

How to Incorporate Children into Labor Supply Equations? An Equivalence-Scale Approach*

Olivier Donni Anderson Vil

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Abstract

We propose a method for modeling the impact of children on labor supply. Building on equivalence scales and collective models, our approach jointly considers the time and monetary costs of children. By imposing restrictions on childcare technology, we identify the total cost of children from labor supply equations with modest data inputs. We apply the model to single mothers from the 2019 PSID. Results suggest labor supply responsiveness to wages is higher for women with children, while the opposite holds for exogenous incomes. We also estimate the total cost of children and the time spent on childcare.

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Keywords: Labor Supply, Collective Model, Price of time, Equivalence scales, Identification, Resource sharing.

*Affiliations: Olivier Donni — THEMA, CY Cergy Paris Université, email: olivier.donni@cyu.fr; Anderson Vil — Paris School of Economics, email: vladerinson@gmail.com. We thank Olivier Bargain, Laurens Cherchye, Marion Goussé, Elena Stancanelli, and Alexandros Theloudis for helpful comments.

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1 Introduction

A substantial body of economic literature has shown that the presence of children significantly affects the labor market behavior of mothers, and to a lesser extent, fathers.¹ As Browning (1992) aptly noted, “typically, any measure of female labor supply (for example, participation or hours if participating) is negatively correlated with any measure of young children (for example, the number of preschool children or the presence of an infant).” Despite this very well-established fact — at least for married women² —, there is no theoretical framework guiding the incorporation of children into labor supply models. In these models, the number of children, sometimes broken down by age and sex, is generally treated as a simple control variable alongside other socio-demographic factors. This stands in stark contrast with the case of demand equations, where a well-developed theoretical framework based on equivalence scales provides a more robust basis for analysis (see Browning, 1992; or Lewbel, 1999, for a survey of the equivalence scales literature).

The primary objective of this paper is to develop a straightforward, theory-consistent method for incorporating children into labor supply models. The key challenge is to capture the impact of children in a parsimonious — yet reasonably realistic — manner. Our approach is designed to enable the effective estimation of labor supply models using traditional workforce surveys, without relying on direct observations of childcare time or monetary costs associated with children. To achieve this, we develop a model of labor supply for single adults, both with and without children, featuring three key elements: (a) both the adult and the child (if present) have distinct preferences, with the adult being altruistic toward the child, (b) the worker’s exogenous incomes are adjusted for the direct monetary costs of raising children (using a function that represents expenditures for children), and (c) the time devoted to childcare is determined by a childcare technology and subtracted from the worker’s total available time. These components collectively define the total cost of children for the parent, encompassing both time and monetary costs. This approach is inspired by recent collective models of consumption, where parents and children are characterized by their own preferences and where resources are shared

¹See Angrist and Evans (1998), Lundberg and Rose (2000), Cools, Markussen, and Strøm (2017), Michaud and Tatsiramos (2011) for a non-exhaustive list.

²As discussed below, the effect of children on single women’s labor supply appears to be more ambiguous, an empirical fact that does not seem to be widely recognized in the literature.

between them (Bargain and Donni, 2012a; Bargain, Donni, and Hentati, 2022; Dunbar, Lewbel, and Pendakur, 2013; Lechene, Pendakur, and Wolff, 2022; Penglase, 2021).³

Our theoretical results can be summarized as follows. First, we demonstrate that workers' preferences and the total cost of children can be identified from the observation of labor supply functions. This is achieved by assuming a relatively general form of childcare technology while the direct monetary cost of children is a linear function of exogenous incomes, a generalization of the proportionality hypothesis used in certain collective demand models (Bargain and Donni, 2012a; Dunbar, Lewbel, and Pendakur, 2013; Lechene, Pendakur, and Wolff, 2022; Penglase, 2021). Second, we introduce a simplified childcare technology that allows the impact of children on labor supply to be captured by three constants that can be estimated. This simplified approach is less data-intensive compared to the more general technology, making it analogous to the incorporation of fixed costs (Cogan, 1981) or to the use of equivalence scales in demand models. More specifically, these constants operate as adjustments to the parent's available time and effective resources, in a manner analogous to Independent-of-the-Base equivalence scales (Blackorby and Donaldson, 1993; Blundell and Lewbel, 1991; Lewbel, 1989, 1991). Our approach stands out as one of the rare efforts to integrate equivalence scales with labor supply models⁴ and, as shown in the Appendix, it can be straightforwardly extended to individuals in couples.

Our contribution can be compared to several studies, many of them relatively dated, that have examined the impact of formal and informal childcare costs on mothers' labor supply decisions (Apps et al., 2016; Averett, Peters, and Waldman, 1997; Baker, Gruber, and Milligan, 2008; Bernal, 2008; Blau and Robins, 1988; Cleveland, Gunderson, and Hyatt, 1996; Connelly, 1992; Doiron, and Kalb, 2005; Kornstad and Thoresen, 2007; Michalopoulos, Robins, and Garfinkel, 1992; Powell, 1997; Ribar, 1992, 1995). These studies, like ours, recognize that the presence of children affects the opportunity cost of parents' time, as the time spent on childcare directly reduces the time available for work, but they differ from ours in that they do not fully account

³See also the foundational work by Bourguignon (1999) for collective models with children and the contributions by Blundell, Chiappori, and Meghir (2005), as well as Cherchye, De Rock, and Vermeulen (2012), which present sophisticated models that require rich data for estimation.

⁴One of notable exception is the labor supply model developed by Hurd and Pencavel (1981), which uses Barten scales to transform prices and wages.

for the income effect associated with children and do not focus on measuring the total cost of children. More importantly, the approaches used in these studies typically require substantially richer data for implementation, notably direct measures of parents' childcare time.

To illustrate our theoretical results, we use a sample of single women, with and without children, drawn from the 2019 Panel Study of Income Dynamics. This sample is well suited to our analytical framework, as it satisfies the core assumptions of the theoretical model and concerns a population that remains largely understudied despite its heightened exposure to poverty risk. Accounting for the labor force participation decision, we estimate a labor supply equation that incorporates the simplified childcare technology. We show that the presence of children increases the sensitivity of labor supply to wages and reduces its sensitivity to exogenous incomes — which is actually the main theoretical prediction of our model. From these estimates, we then compute the total cost of children by valuing time at the market wage for working mothers and at an estimated shadow price for non-working mothers.

The remainder of this paper is as follows. Section 2 presents the model. Section 3 specifies the data and the estimation method. Section 4 discusses the estimation results. Section 5 concludes.

2 The Model

Our analysis adopts a static model of labor supply and focuses on a single woman (or more broadly a single adult) with or without children. An extension to individuals in couples is suggested in Appendix.

2.1 The General Model

A. The case of a woman with children. We begin by examining the case of a woman with children. Her labor time is represented by h and her childcare time by t , while c denotes her consumption of a composite good. In addition, the consumption of children is denoted by m and the number of children, assumed to be exogenously given, by n . To capture her preferences, we assume a utility function of the form:

$$u_f(h + t, c) + \lambda(n)u_c(m), \tag{1}$$

where $u_f(\cdot)$ denotes the woman's sub-utility, $u_c(\cdot)$ denotes the children's sub-utility, and $\lambda(n)$ is a weight that represents mothers' altruism. Specifically, the woman's sub-utility decreases with total labor supply but increases with consumption, while the children's sub-utility increases with their consumption. Both sub-utilities are strictly concave in their respective arguments.

Before proceeding further, we would like to make a few remarks. Firstly, the mother's caregiving behavior is consistent with Becker's altruism (or 'caring'), as commonly employed in the literature on collective models. Secondly, the mother's utility function is additive. While this may appear restrictive, our results remain valid even with a more general form of separability in the utility function, such as $U(u_f(h + t, c), u_c(m), \lambda(n))$ for some aggregating function $U(\cdot)$. Thirdly, the children's sub-utility reflects the utility of an average or representative child. Finally, labor and childcare time are assumed to be perfectly substitutable, which is perhaps the most restrictive assumption in our model. Despite this, such an assumption is frequently employed in the literature for its practical convenience (Gronau, 1977; Donni, 2008; Donni and Matteazzi, 2012) and is crucial in our framework.⁵

The household budget constraint is given by

$$y + hw \geq c + m + k \tag{2}$$

where w denotes the woman's wage rate, y her other exogenous incomes, and k her expenditures on childcare time (i.e., the cost of purchasing daycare services). The prices of both parent's and children's consumption are normalized to unity. The childcare technology constraint is given by

$$f(t, n) + k \geq T_c(n), \tag{3}$$

where $T_c(n)$ can be interpreted as the amount of childcare time required for the n children, and $f(t, n)$ is a function that is increasing and concave in t . To start, let us consider an extreme case where $T_c(n)$ represents 24 hours per day for a single child. In that case, the mother thus has to allocate 24 hours to childcare, either by providing it herself or by purchasing it from the market.

⁵The main consequence of this assumption is that the price of time spent on childcare coincides with the market wage, i.e., the price of time spent on leisure. The case where these prices of time are distinct is explored in a companion paper (Donni and Vil, 2025).

If the mother's time and market-based childcare are perfectly substitutable, this constraint can be expressed as $t + k \geq 24$ where k represents the cost of purchased formal childcare, assuming the price per hour is normalized to one. Although intuitive, however, this interpretation is overly restrictive. First, childcare arrangements may involve inefficiencies related to transportation or administrative tasks. Second, in-home childcare may exhibit diminishing returns due to parental fatigue. Third, the quality of care provided at home may differ from that available through formal childcare services. More realistically, we thus assume imperfect substitutability between the mother's time and market-based childcare.

The optimization problem of the mother can thus be represented as:

$$\max_{h,t,m,c,k} u_f(h + t, c) + \lambda(n)u_c(m) \quad (4)$$

subject to the budget constraint (2), the childcare technology constraint (3), and non-negativity constraints. For an interior solution, this problem can be decomposed into three stages.⁶ In the first stage, the mother chooses the optimal combination of her childcare time and market childcare time in order to minimize the total cost:

$$e_t(w, n) = \min_{t,k} (tw + k) \text{ subject to } f(t, n) + k \geq T_c(n),$$

where $e_t(w, n)$ is the time cost of children. In the second stage, she also chooses the optimal level of expenditures m for children, denoted as $m = e_m(y, w, n)$, in order to solve the following optimization problem:

$$\max_{e_m} v_f(w, y - e_t(w, n) - e_m) + \lambda(n)u_c(e_m). \quad (5)$$

taking $e_t(w, n)$ as given, where $v_f(\cdot)$ is the indirect utility function corresponding to $u_f(\cdot)$. In the third stage, she maximizes her sub-utility taking $e_t(w, n)$, $e_m(y, w, n)$ and $t(w, n)$ as given. The optimization problem is:

$$\max_{h,c} u_f(h + t(w, n), c) \text{ subject to } y + (h + t(w, n))w \geq c + e_m(y, w, n) + e_t(w, n). \quad (6)$$

⁶This decomposition follows from a separation principle, well-known in agricultural economics, that is closely related to the perfect substitutability between childcare time and labor time in the parent's preferences.

The solution of this optimization problem is of the form:

$$h(y, w, n) = F(w, y - e_t(w, n) - e_m(y, w, n)) - t(w, n), \quad (7)$$

where $F(\cdot)$ is the baseline labor supply function that follows from the woman's utility only — it is a traditional Marshallian labor supply function that satisfies Slutsky conditions. Structurally, the functional form (7) is analogous to a labor supply model that incorporates both a fixed monetary cost and a fixed time cost, as outlined by Cogan (1980). These fixed costs influence working time in two primary ways: by shifting the baseline labor supply function and by shifting the mother's available resources. These two effects operate differently: the first tends to reduce the hours worked, while the second tends to increase them through an income effect (if leisure is normal). Consequently, the theoretical impact of children on working time is indeterminate.⁷

B. The case of a woman without children. We now briefly consider the case of a childless woman and we assume that the utility is of the form:

$$u_f = \psi(u_f(h, c))$$

where $\psi(\cdot)$ is an increasing transformation of the sub-utility of women with children. Stated differently, the marginal rate of substitution between labor and consumption is identical for women, regardless of whether they have children or not. If her utility function is maximized under the budget constraint $y + hw \geq c$ and non-negativity constraints, then her labor supply function is of the form:

$$h(y, w, n = 0) = F(w, y). \quad (8)$$

Our identification strategy is based on the idea that the function $F(\cdot)$ is the same for women, regardless of whether they have children or not, as assumed by Bargain and Donni (2012a) or Bargain, Donni and Hentati (2022). However, additional structure is necessary.

⁷The first effect is likely to dominate for married women, as suggested by the numerous studies cited in the introduction. The second effect may help explain why married men often increase their working hours when children are present. As we shall show, these two effects tend to offset each other for single women, implying that children have a more ambiguous impact on labor supply at either the intensive or extensive margin.

In several recent contributions to collective models of consumption (Bargain and Donni, 2012a; Dunbar, Lewbel and Pendakur, 2013; Lechene, Pendakur and Wolff, 2022; Penglase, 2021), the assumption that the cost of children is proportional to total household income is frequently employed. Menon, Pendakur and Perali (2012) provide evidence using disaggregated data that this assumption holds for children's expenditures. For labor supply models, nonetheless, this assumption seems to be less appropriate. In consumption models, total household income includes both labor and non-labor incomes. Consequently, proportionality in this context implies that $e_m = b(w, n) \cdot (h(w, n) \cdot w + y) = a(w, n) + b(w, n) \cdot y$ where $a(w, n) = b(w, n) \cdot h(w, n) \cdot w$. The linearity property for labor supply models can thus be seen as a natural generalization of the proportionality property used in consumption models.

Assumption L (Linearity). Expenditures on the childcare good is a linear function of exogenous incomes. That is, $e_m(y, w, n) = a_m(w, n) + y \cdot b_m(w, n)$, where $a_m(w, n)$ and $b_m(w, n)$ are functions, with $b_m(w, n) \in (0, 1)$.

Bargain and Donni (2012b) demonstrate that linearity is automatically achieved when the indirect utility functions corresponding to $u_f(\cdot)$ and $u_c(\cdot)$ in (1) are of the Constant Absolute Elasticity of Substitution (CARA) form. The constant $a_m(w, n)$ is then a linear function of the degree of altruism $\lambda(n)$.

Under this assumption, it can be shown that the main structural components of the model are identified. We first define the total cost of children as the sum of its time and monetary components: $e(y, w, n) = e_m(y, w, n) + e_t(w, n)$.

Proposition. Assume L. Let $h(y, w, n)$ be market labor supply, observed for women with and without children. For any (y, w, n) such that $h(y, w, n) > 0$, we have:

- a) If $h'_y \neq 0$ and $h''_y \neq 0$, then, under regularity conditions, the total cost of children $e(y, w, n)$ is identified.
- b) If $h'_y \neq 0$ and $h''_y = 0$, then the function $b_m(w, n)$ is identified.

Proof. The function $F(\cdot)$ can be recovered from the observed market-time behavior of childless women. Differentiating the labor supply function with respect to y yields

$$h'_y(y, w, n) = F'_2(w, y - e_t(w, n) - a_m(w, n) - y b_m(w, n)) (1 - b_m(w, n)), \quad (9)$$

where the left-hand side is observed, and F'_2 denotes the partial derivative of $F(\cdot)$ with respect to its second argument. We now distinguish two cases.

(i) Linear case. Suppose that the baseline labor supply function is linear in y , i.e.,

$$h(y, w, n) = \alpha(w) + \beta(w) \left(y - e_t(w, n) - a_m(w, n) - y b_m(w, n) \right) - t(w, n), \quad (10)$$

where $\alpha(w)$ and $\beta(w)$ are functions of w only. Thus, equation (9) becomes:

$$h'_y(y, w, n) = \beta(w) (1 - b_m(w, n)),$$

which immediately gives:

$$b_m(w, n) = 1 - \frac{h'_y(y, w, n)}{\beta(w)}.$$

The remaining components of the total cost, however, cannot be separately identified. Indeed, if a cost function $e(y, w, n)$ is compatible with observed labor supply behavior, then

$$e^*(y, w, n) = e(y, w, n) + k(w, n),$$

for any function $k(w, n)$, is also compatible. To see this, consider the observationally equivalent representation of the labor supply equation:

$$h(y, w, n) = \alpha(w) + \beta(w)(y - e(y, w, n) - \Delta(w, n)) - (t(w, n) - \beta(w)\Delta(w, n)). \quad (11)$$

In this equation, the total cost function is given by $e^*(y, w, n) = e(y, w, n) + \Delta(w, n)$. The latter can be decomposed into a time cost component and a monetary cost component, defined

respectively by

$$e_t^*(w, n) = e_t(w, n) - \int^w \beta(r) \Delta(r, n) dr,$$

$$e_m^*(y, w, n) = e_m(y, w, n) + \Delta(w, n) + \int^w \beta(r) \Delta(r, n) dr.$$

The associated childcare time demand is given by

$$t^*(w, n) = t(w, n) - \beta(w) \Delta(w, n).$$

The alternative representation of the labor supply equation (11) is thus totally consistent with our theoretical assumptions.

(ii) General case. The baseline labor supply function is not linear in y . Thus, equation (9) becomes:

$$\frac{h'_y(w, y, n)}{1 - b_m(w, n)} = F'_2(w, y - e_t(w, n) - a_m(w, n) - y \cdot b_m(w, n)). \quad (12)$$

For each fixed w , assume that the map $s \mapsto F'_2(w, s)$ is one-to-one on the relevant domain. Let $G'_2(w, \cdot) = [F'_2(w, \cdot)]^{-1}$ be its inverse with respect to the second argument, so that $G'_2(w, F'_2(w, s)) = s$. Applying this inverse to equation (12) gives:

$$G'_2 \left(w, \frac{h'_y(w, y, n)}{1 - b_m(w, n)} \right) = y - e_t(w, n) - a_m(w, n) - y \cdot b_m(w, n). \quad (13)$$

Differentiating (13) with respect to y gives:

$$G''_2 \left(w, \frac{h'_y(w, y, n)}{1 - b_m(w, n)} \right) \times h''_y(w, y, n) = (1 - b_m(w, n))^2, \quad (14)$$

where $G''_2 = 1/F'_2$. Under regularity conditions, this equation defines $b_m(w, n)$. More precisely, if the following condition is satisfied for any (w, n) ,

$$\left| \frac{1}{2} G'''_2 \left(w, \frac{h'_y(w, y, n)}{1 - b(w, n)} \right) \times \frac{h'_y(w, y, n) \times h''_y(w, y, n)}{(1 - b(w, n))^3} \right| < 1,$$

where $G'''_2 = -F'''_2/(F'_2)^3$, then the equation is a contraction with respect to $(1 - b_m(w, n))^2$, ensuring that $b_m(w, n)$ is uniquely defined for any (w, n) , with $b_m(w, n) \in (0, 1)$. This condition

is sufficient, though not necessary.

Once $b_m(w, n)$ is defined, substituting its value back into expression (13) gives $e_t(w, n) + a_m(w, n)$ and therefore $e(y, w, n)$. Finally, from expression (7), we recover the childcare time function. $\square$

This result shows that the total cost of children can be identified without needing information on childcare time or parental expenditures on children. Although the decomposition of the total cost into what we define as a monetary component $e_m(w, n)$ and a time component $e_t(w, n)$ is not separately identifiable, it is nonetheless possible to recover, on the one hand, the value of a mother's time devoted to children, given by $wt(w, n)$, and, on the other hand, the value of purchased goods and services for children, given by $e(y, w, n) - wt(w, n)$.

Identification of the total cost relies on the assumptions that the non-negativity constraints are not binding (i.e., the woman is participating in the labor market) and that the baseline labor supply function is non-linear in exogenous incomes. When the function is instead linear, the cost of children is only partially identified. The main challenge then is empirical: identification requires precise estimation of second-order derivatives of labor supply, which is difficult with standard micro data. An additional complication comes from non-working women. For women who do not work, we can assume that the childcare technology $f(t, n)$ is the same as for working women, so identification ultimately rests on the behavior of the latter. A more serious issue, however, is that the function $e_m(y, w, n)$ is not identifiable on the set $Q = \{(y, w, n) : h(y, w, n) = 0\}$. Extending identification from the participation region $P = \{(y, w, n) : h(y, w, n) > 0\}$ to the non-participation region Q therefore requires stronger parametric assumptions. Finally, even if such parametric assumptions are imposed, the wage rate no longer provides a valid measure of the price of time. Instead, the relevant shadow price can be recovered by inverting the labor supply function (Heckman, 1974; Neary and Roberts, 1980; Kooreman and Kapteyn, 1986). We return to this issue in the following sections.

2.2 A More Tractable Model

To obtain a more tractable framework, we introduce a simple specification for childcare technology. Specifically, we assume that the childcare cost function is linear in w , expressed as:

$$e_t(w, n) = a_t(n) + b_t(n)w$$

where $a_t(n)$ and $b_t(n)$ are functions of n only. The underlying childcare technology can be described as follows:

$$f(t, n) = \begin{cases} T_c(n) - a_t(n) & \text{if } t \geq b_t(n), \\ -\infty & \text{if } t < b_t(n). \end{cases}$$

Intuitively, the mother has a fixed amount of time she can devote to childcare, and she cannot reduce this below a certain threshold regardless of her wage. Childcare time is therefore perfectly rigid, with $t(w, n) = b_t(n)$. This is a restrictive assumption, but it is broadly consistent with empirical evidence suggesting that childcare time is not very responsive to wage rates (Bloemen and Stancanelli, 2014, for France; Kalenkoski, Ribar, Stratton, 2009, for U.K.; Donni and Vil, 2025, and Kimmel and Connelly, 2007, for U.S., Hallberg and Klevmarken, 2003, and Gustafsson and Kjullin, 1994, for Sweden).⁸ Similarly, the expenditures for children is simplified as follows:

$$e_m(y, n) = a_m(n) + b_m(n)y.$$

This remains a relatively mild simplification compared with Assumption L, which is supported, at least in part, by the empirical results discussed in the previous subsection.

Incorporating the childcare time function and the childcare expenditure function into the labor supply function, we obtain:

$$h(y, w, n) = F(w, y - a(n) - b_t(n)w - b_m(n)y) - b_t(n). \quad (15)$$

where $a(n) = a_t(n) + a_m(n)$. This transformation of the basis labor supply function is similar to the double translation of demand functions in the case of Independent-of-the-Base equivalence scales (Lewbel, 1989, 1991; Blundell and Lewbel, 1991; Blackorby and Donaldson, 1993). If $F'_1 > 0$ and $F'_2 < 0$, as can be generally expected, with $b_t(n) > 0$ and $b_m(n) > 0$, *the presence of children increases the sensitivity of labor supply to wages and reduces its sensitivity to exogenous*

⁸These studies focus on couples. It is plausible that this effect is even weaker for single mothers, who lack the intra-household margins of adjustment available within couples.

incomes.⁹ This property constitutes a testable restriction of the model.¹⁰

Note finally that, if the assumptions on the cost functions hold for all women, identification obtained on $P = \{(y, x, n) : h(y, x, n) > 0\}$ can be extended to $Q = \{(y, x, n) : h(y, x, n) = 0\}$.

3 Data and Estimation Method

3.1 The Data Set

Our estimation is based on the widely used Panel Study of Income Dynamics (PSID). The PSID is a longitudinal survey of the University of Michigan that reports income, employment trajectories, and other variables over time. The dataset provides extensive information on housing, employment, income, wealth, as well as detailed individual and family characteristics. Unlike earlier waves, the 2019 wave also contains household expenditures for various goods and services. This information is used to construct the variable “net total expenditures”, which we employ in the empirical analysis in place of exogenous incomes. This variable is calculated by subtracting labor income from household total expenditures. As discussed below, it may be endogenous to labor supply decisions, an issue that we explicitly address in the empirical strategy.

The initial sample consisted of 8,195 households. Our primary focus is on single women, both with and without children, aged 20 to 60. Although our framework can be extended to couples, restricting attention to single women allows us to sidestep the additional complexities that arise when modeling couples, particularly due to the limited flexibility in men’s labor supply. Moreover, focusing on single women offers significant value, as they represent a group frequently

⁹If the monetary cost of children varies with the wage rate, the impact of wages on hours may become ambiguous.

¹⁰The greater sensitivity of mothers’ working hours to wage rates has been documented in several empirical studies: for instance, Blundell et al. (2016) and Meyer and Rosenbaum (2001) for single mothers in the U.K. and U.S., respectively, and Blau and Kahn (2007, 2013) for married mothers in the U.S.

Table 1: Descriptive Statistics

		All Women	Non-W. Women	Working Women	Childless Women	Single Mothers
Labor market participation	mean	0.85	0.00	1.00	0.83	0.90
	std	0.36	0.00	0.00	0.38	0.30
Annual hours worked	mean	1,634.41	0.00	1,932.68	1,638.58	1,619.86
	std	980.81	0.00	748.80	1,003.99	896.80
Hourly wages	mean	17.54	0.00	20.74	18.78	13.22
	std	16.91	0.00	16.48	18.14	10.56
Net total expenditures	mean	-0.23	20.71	-4.05	-3.01	9.47
	std	27.41	14.29	27.51	29.24	16.41
Age	mean	40.24	47.53	38.91	42.19	33.44
	std	11.76	10.90	11.43	11.70	9.19
Years of education	mean	13.64	12.18	13.91	13.85	12.90
	std	2.40	2.23	2.33	2.36	2.40
Black	mean	0.60	0.70	0.58	0.58	0.68
	std	0.49	0.46	0.49	0.49	0.47
Hispanic	mean	0.09	0.05	0.09	0.07	0.13
	std	0.28	0.22	0.29	0.26	0.34
Number of children	mean	0.35	0.24	0.36	0.00	1.55
	std	0.69	0.62	0.70	0.00	0.50
Sample Size		1270	196	1074	987	283

overlooked in research, despite being particularly vulnerable to poverty.¹¹ Finally, for single mothers, we further restricted our sample to those with no more than two children. Given that our theoretical model accounts for daycare expenditures, we excluded households where the youngest child was older than six. After eliminating observations with relevant missing data, our final sample consisted of 1,270 households, with 987 single women and 283 single mothers.

In Table 1, we present summary statistics for the key variables employed in this research. On average, single women work 1,634 hours annually, with a participation rate of approximately 85%, showing slight variations based on whether they have children. Interestingly, single moth-

¹¹Empirical studies that focus explicitly on the labor supply of single women are relatively scarce. Bishop, Heim, and Mihaly (2009) provide one of the few systematic analyses of single women’s labor supply behavior. Kimmel and Kniesner (1998) compare labor supply responses across marital status groups. Admittedly, there is a much larger body of research (Eissa and Liebman (1996) or Meyer and Rosenbaum (2001) for the Earned Income Tax Credit in the United States and Blundell et al. (2016) for the Working Families Tax Credit in the United Kingdom, among many others) that examines the labor supply effects of tax and transfer policies that disproportionately or explicitly target single women — especially lone mothers — but these studies are not designed to estimate a full structural labor supply model for single women.

ers have a higher participation rate than their childless counterparts, despite earning a lower hourly wage.¹² This evidence suggests that the income effect associated with having children is substantial for single mothers and may outweigh the substitution effect. Patterns in the net total expenditures reinforce this picture. While childless women generally manage to cover their expenses with their earnings, single mothers more often find that their wages fall short of meeting their household's total expenditures.¹³ At the same time, however, single mothers work slightly fewer annual hours than childless women, highlighting the complexity of the relationship between children and labor supply, which cannot be adequately captured by a simple control variable.

3.2 Empirical Specification

For the baseline labor supply function, we follow Brown, Levin, and Ulph (1976) and adopt a quadratic specification:

$$h = \alpha_0 z_h + \alpha_1 w + \alpha_2 y + \alpha_3 w^2 + \alpha_4 y^2 + \alpha_5 yw + u \quad (16)$$

where $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are parameters, u is an error term, and z_h are additional control variables. These control variables include a constant, age, age squared, and indicators for Black and Hispanic women. Because of its flexibility, this specification does not impose global normality of leisure, nor does it guarantee global satisfaction of the Slutsky positivity condition. However, this need not be a concern in practice, as these regularity conditions may hold over a sufficiently large domain of economic relevance.

For the childcare technology, we assume the following. (i) The time cost of children is of the form:

$$e_t(w) = (a_t + b_t w) \times \delta(n), \quad (17)$$

where a_t and $b_t > 0$ are constants and $\delta(n)$ is a scaling function depending on the number of

¹²As noted earlier, this sharply contrasts with the well-documented pattern among women in couples, for whom those with children exhibit substantially lower participation rates than their childless counterparts.

¹³The expenditures reported in the PSID are not exhaustive and do not cover the full range of household expenditures.

children which captures economies of scale. This function satisfies $\delta(0) = 0$ and, for normalization, $\delta(1) = 1$; hence, we have chosen: $\delta(n) = \sqrt{n}$. From Shephard's Lemma, we obtain: $t(w, n) = b_t \times \delta(n)$. (ii) The monetary cost of children is specified as:

$$e_m(y, w) = (a_m + b_m y) \times \delta(n), \quad (18)$$

where a_m and b_m are constants. The constants a_t and a_m cannot be identified separately but their sum $a = a_t + a_m$ can be identified.

The quadratic form encompasses the Hausman (1980)'s linear specification as a special case with appealing properties. Specifically, it can be written as:

$$h(w, y, n) = \alpha_0 z_h + \alpha_1 w + \alpha_2 (y - a\delta(n) - b_t \delta(n)w - b_m \delta(n)y) - b_t \delta(n) + u \quad (19)$$

or

$$h(w, y, n) = \alpha_0^*(z_h, n) + \alpha_1^*(n)w + \alpha_2^*(n)y + u \quad (20)$$

where

$$\begin{aligned} \alpha_0^*(n) &= \alpha_0 z - \alpha_2 a \delta(n) - b_t \delta(n) \\ \alpha_1^*(n) &= \alpha_1 - \alpha_2 b_t \delta(n) \\ \alpha_2^*(n) &= \alpha_2 (1 - b_m \delta(n)). \end{aligned} \quad (21)$$

The parameters of the linear baseline labor supply equation are easier to interpret. The presence of children shifts the intercept and alters both the wage effect and the effect of net total expenditures, while preserving the linear structure of the labor supply equation. Note also that, despite the fact that the function is linear, the total cost of children remains identifiable.

3.3 Estimation Method

To estimate the model, we adopt a control function approach within a likelihood framework. We begin by specifying a market wage equation to account for the presence of non-working

women and for the endogeneity of hourly wages:

$$w = \beta_w z_w + \sigma_\nu \nu \quad (22)$$

where β_w and σ_ν are parameters, z_w are control variables, and ν is a disturbance with zero mean and unit variance. Control variables include the variables in z_h , and the number of years of education.

A further endogeneity issue concerns net total expenditures. For example, measurement error in expenditures or earnings may influence the decision to work more or fewer hours, thereby inducing a correlation between net total expenditures and the disturbance term. To correct for this, we incorporate into the labor supply equation the residuals obtained from a first-stage regression:

$$y = \beta_y z_y + \sigma_\mu \mu \quad (23)$$

where β_y and σ_μ are parameters, z_y are instrumental variables, and μ is a disturbance with zero mean and unit variance. Instrumental variables include the variables in z_w and the various sources of household income, which constitute exogenous determinants of net total expenditures (Blundell and MaCurdy, 1999).

The main parameters are estimated by maximizing the log-likelihood function. To do so, the disturbance term in the labor supply equation is specified as $u = \rho_\mu \mu + \rho_\nu \nu + \sigma_\epsilon \epsilon$, where ρ_μ , ρ_ν and σ_ϵ are parameters while ϵ is a disturbance with zero mean and unit variance. We assume that both ν and ϵ are normally distributed, and we denote by $\hat{\mu}$ the residual obtained from the estimation of equation (23). For notational convenience, we rewrite the labor supply equation in compact form as $h = h_0(w) + \rho_\mu \mu + \rho_\nu \nu + \sigma_\epsilon \epsilon$, and the wage equations as $w = w_0 + \sigma_\nu \nu$, where $h_0(w)$ and w_0 have straightforward interpretations. The woman's decision problem can then be expressed as:

$$\begin{aligned} h &= h_0(w) + \rho_\mu \mu + \rho_\nu \frac{w - w_0}{\sigma_\nu} + \sigma_\epsilon \epsilon, & \text{if } \epsilon > -\frac{h_0(w) + \rho_\mu \mu + \rho_\nu (w - w_0)/\sigma_\nu}{\sigma_\epsilon} \\ w &= w_0 + \sigma_\nu \nu \end{aligned} \quad (24)$$

and

$$h = 0 \quad \text{if } \varepsilon \leq -\frac{h_0(w_0 + \sigma_\nu \nu) + \rho_\mu \mu + \rho_\nu \nu}{\sigma_\varepsilon} \quad (25)$$

Substituting μ with $\hat{\mu}$, the contribution to likelihood for women who do not work is:

$$\int_{-\infty}^{+\infty} \Phi \left(-\frac{h_0(w_0 + \sigma_\nu \nu) + \rho_\mu \hat{\mu} + \rho_\nu \nu}{\sigma_\varepsilon} \right) \phi(\nu) \cdot d\nu$$

where Φ and ϕ are, respectively, the cumulative probability function and the density probability function of the standardized normal distribution. The contribution to likelihood for women who do work is:

$$\frac{1}{\sigma_\varepsilon \sigma_\nu} \cdot \phi \left(\frac{w - w_0}{\sigma_\nu} \right) \cdot \phi \left(\frac{h - h_0(w) - \rho_\mu \hat{\mu} - \rho_\nu (w - w_0)/\sigma_\nu}{\sigma_\varepsilon} \right).$$

Summing the logarithm of these expressions over all observations gives the log-likelihood function, which is maximized to estimate all the parameters, including ρ_ν , ρ_μ , σ_ν and σ_ε .

3.4 The Cost of Children

The total cost of children can be computed from the estimated parameters. To do this, we must distinguish between whether the mother is working or not. For working mothers, the total cost of children is simply given by:

$$e_t(w) + e_m(y) = (a + b_t w + b_m y) \times \delta(n), \quad (26)$$

where the wage rate serves as a measure of the price of time. For nonworking mothers, however, the wage rate can no longer be considered as a measure of the price of time. Instead, we can use the reservation wage, denoted as w_R , obtained by inverting the labor supply equation evaluated at $h = 0$. Under our quadratic specification, the resulting equation to be solved takes the form:

$$Aw_R^2 + Bw_R + C = 0$$

where

$$\begin{aligned}
A &= \alpha_2 + \alpha_4 b_t^2 \delta(n)^2 - \alpha_5 b_t \delta(n) \\
B &= \alpha_1 - \alpha_3 b_t \delta(n) - 2\alpha_4 (y - (a + b_m y) \delta(n)) b_t \delta(n) + \alpha_5 (y - (a + b_m y) \delta(n)) \\
C &= \alpha_0 z_h + \alpha_3 (y - (a + b_m y) \delta(n)) + \alpha_4 (y - (a + b_m y) \delta(n))^2 - b_t \delta(n) + \rho_\mu \mu + \rho_\nu \nu + \sigma_\varepsilon \varepsilon
\end{aligned}$$

If $B^2 - 4AC > 0$, this equation has two solutions for w_R . Only one of these solutions satisfies the Slutsky positivity condition (which ensures that the slope of the labor supply function along the participation frontier is positive). This solution is defined as:

$$w_R = \frac{-B + \sqrt{B^2 - 4AC}}{2A}.$$

If $A < 0$ and $B > 0$, which seems reasonable if non-linearity is moderate, the price of time is positive.

Since C depends on disturbances μ , ν and ε , which can vary from $-\infty$ to $+\infty$, there will inevitably be instances where $B^2 - 4AC < 0$ for some individuals, making the price of time impossible to compute. While this may not be a major concern, as such instances are likely to be rare, it does mean that the moments of the probability distribution of the price of time cannot be reliably calculated. Nevertheless, we can still estimate certain aspects of this probability distribution, such as quantiles, to gain insight into its variability.

4 Estimation Results and Discussion

In this section, we present the main results for the baseline model along with those for four alternative specifications. We begin by presenting the estimates from the preliminary net total expenditures regression, followed by the maximum likelihood estimates of the structural model and the implications of our results for the cost of children.

4.1 Endogeneity: A Preliminary Regression

The parameter estimates of the net total expenditures equation are reported in Table 2. We find that, with the exception of total asset income and its square, all sources of income have a statistically significant effect.

Table 2: Estimated Parameters of the Net Total Expenditures Equation

Net Total Expenditures Equation		
Intercept	64.171	(10.968)
Years of education	-2.927	(0.310)
Age	-1.558	(0.547)
Age squared	1.707	(0.663)
Black	4.669	(1.550)
Hispanic	1.811	(2.659)
Total asset income	-0.258	(0.309)
Total welfare income	1.540	(0.379)
Total transfer income	1.982	(0.418)
Total other income	0.439	(0.052)
Total asset income squared	0.001	(0.003)
Total welfare income squared	-0.025	(0.012)
Total transfer income squared	-0.034	(0.014)
Total other income squared	-0.002	(0.000)

Notes: Standard errors in parentheses. Household net total expenditures are in thousands of dollars.

4.2 The ML Parameter Estimates

The estimated parameters of the baseline labor supply, child cost and wage equations are reported in Table 3 for four alternative specifications. Specification LIN corresponds to a linear baseline labor supply equation (Hausman). Specification QUA corresponds a quadratic baseline labor supply equation (Brown-Levin-Ulph). Specification QUA-R is identical to specification QUA but is estimated on a subsample in which the top 5% of hourly wage and market hours observations have been trimmed (which corresponds to 126 observations). This robustness check is intended to assess the sensitivity of our estimates to extreme observations.¹⁴ Specification QUA-W is identical to specification QUA but the sample is restricted to working women only. This restriction helps isolate the intensive margin of labor supply, abstracting from participation decisions that may be influenced by involuntary unemployment or fixed costs unrelated to children. All these parameter estimates are then complemented by simulation results reported in Tables 4 and 5. These simulations are used to evaluate, at different quartiles of the distribution, the marginal effects of wages and net total expenditures on working hours, as well as to compute the price of time, the market wage and the cost of children. They are conducted with the parameter estimates from specification QUA by drawing 200 realizations of the error terms ν

¹⁴Other trimming rules were explored and yielded similar qualitative conclusions.

Table 3: Estimated parameters of the baseline labor supply, child cost and wage equations

	LIN		QUA		QUA-R		QUA-W	
Labor Supply Equation								
Intercept	0.900	(0.267)	0.981	(0.340)	0.010	(0.296)	1.996	(0.111)
Age	0.012	(0.383)	0.135	(0.481)	-0.084	(0.336)	0.189	(0.196)
Age squared	-0.318	(0.363)	-0.480	(0.444)	-0.213	(0.320)	-0.198	(0.193)
Black	0.091	(0.050)	0.102	(0.060)	0.104	(0.045)	0.037	(0.027)
Hispanic	0.071	(0.041)	0.058	(0.048)	0.091	(0.039)	0.025	(0.024)
w	0.027	(0.012)	0.015	(0.015)	0.123	(0.017)	-0.007	(0.006)
y	-0.028	(0.003)	-0.031	(0.003)	-0.015	(0.004)	-0.023	(0.003)
$w^2 \times 10^{-2}$	—	—	0.026	(0.004)	-0.210	(0.024)	0.004	(0.006)
$y^2 \times 10^{-2}$	—	—	-0.001	(0.001)	0.011	(0.003)	0.002	(0.001)
$wy \times 10^{-2}$	—	—	0.011	(0.004)	-0.064	(0.011)	0.014	(0.005)
$\hat{\mu}$	0.005	(0.003)	-0.002	(0.003)	-0.001	(0.003)	0.004	(0.003)
Child Cost Equation								
Intercept	5.016	(2.810)	10.513	(4.759)	7.858	(2.230)	29.216	(12.150)
w	0.085	(0.119)	0.497	(0.178)	0.190	(0.086)	0.992	(0.295)
y	-0.095	(0.092)	0.089	(0.084)	0.174	(0.095)	0.209	(0.107)
Wage Equation								
Intercept	22.864	(0.702)	23.172	(0.660)	19.824	(0.377)	20.738	(0.720)
Years of education	3.811	(0.563)	3.533	(0.557)	2.730	(0.336)	5.114	(0.516)
Age	7.666	(4.899)	7.703	(5.120)	7.337	(3.175)	8.895	(4.547)
Age squared	-3.354	(4.811)	-3.250	(4.932)	-3.872	(3.139)	-6.006	(4.413)
Black	-2.011	(0.574)	-1.967	(0.553)	-1.474	(0.356)	-2.180	(0.568)
Hispanic	0.071	(0.492)	0.012	(0.549)	-0.441	(0.376)	0.410	(0.557)
Objective Function	-6049.002		-5986.194		-4751.902		-5528.266	

Notes: Standard errors are in parenthesis. LIN = Linear baseline labor supply equation (Hausman). QUA = Quadratic baseline labor supply equation (Brown-Levin-Ulph). QUA-R = Same as QUA estimated on a subsample in which extreme values have been trimmed. QUA-W = Same as QUA estimated on a subsample containing only working women. In all models, annual working hours are measured in units of 1,000 hours and household net total expenditures are in thousands of dollars.

and ϵ from their assumed normal distributions for each observation using Halton sequences. The bottom panel of Table 3 reports the estimated coefficients of the wage equation. The results are stable across all specifications and broadly consistent with the existing literature. Importantly, the years of education (which serve as the excluded instrument for wage rates) are highly significant and exhibit the expected positive sign.

The top panel of the table presents the estimated parameters of the baseline labor supply equation. In these estimations, annual working hours are measured in units of 1,000 hours.

For both the linear and quadratic specifications, the coefficient on net total expenditures is statistically significant. For the linear specification, a \$1,000 increase in net total expenditures is associated with a reduction of approximately 28 annual working hours for a childless woman. In the quadratic specification estimated on the full sample, the coefficient on the interaction term between the wage rate and net total expenditures is statistically significant, whereas the coefficient on the squared term is not. Moreover, simulations of the marginal effect of net total expenditures for the quadratic specification in Table 4 indicate an effect of comparable magnitude to those obtained under the linear specification. Specifically, for a childless working woman, the implied reduction in annual working hours ranges from about 29.3 hours at the first quartile to about 26.5 hours at the third quartile with the full sample. The results obtained from the restricted sample and from the sample of working women only differ noticeably from those for the full sample, as the coefficient on net total expenditures becomes statistically significant in both cases, although its magnitude remains small. Unreported results further indicate that the marginal effects of net total expenditures in the restricted sample are very similar to those obtained in the full sample, whereas the corresponding effects in the sample of working women only are somewhat smaller.¹⁵ Taken together, these findings suggest that net total expenditures exert only a moderately nonlinear effect on working hours. As a consequence, the general cost function is unlikely to be estimable with sufficient reliability given our data.

Our findings also imply that wage rates have a substantial effect on working hours. In the linear specification, the wage coefficient is statistically significant and economically large: a \$10 increase in the hourly wage for a childless woman is associated with an increase of approximately 270 annual working hours. In the quadratic specification, the effect of wages on working hours is markedly nonlinear: both the coefficient on the squared wage term and, as already mentioned, the interaction term are statistically significant. When evaluated at the quartiles of the distribution, a \$10 wage increase for a childless, working woman implies an increase in annual working hours ranging from about 207 to 328 hours for the full sample, as shown in Table 4. Notably, the results obtained from the full sample and the alternative samples differ substantially, with an even more pronounced curvature of labor supply in the case of the restricted sample. Consistent with this evidence, a likelihood ratio test strongly

¹⁵Small differences between the results obtained with the full and the alternative samples were anticipated, since the selection of observations is based on endogenous variables.

Table 4: Quartiles of the distributions marginal effects, prices of time and hourly wages, computed with the quadratic specification and the full sample (QUA)

		W./Non-w. women			Non-w. women			Working women		
		25%	50%	75%	25%	50%	75%	25%	50%	75%
Childless women and mothers	Marginal effect of w	17.4	27.3	35.8	0.0	0.0	0.0	22.7	29.8	37.3
	Marginal effect of y	-28.6	-26.8	-24.3	0.0	0.0	0.0	-28.9	-27.3	-25.7
	Price of time	13.4	25.7	38.8	30.9	46.8	64.1	11.6	23.2	34.7
	Market wage	11.7	23.2	34.7	12.0	23.4	34.7	11.6	23.2	34.7
Childless women	Marginal effect of w	15.1	24.4	31.4	0.0	0.0	0.0	20.7	26.7	32.8
	Marginal effect of y	-29.0	-27.4	-25.2	0.0	0.0	0.0	-29.3	-27.9	-26.5
	Price of time	14.9	27.2	40.4	32.8	48.7	65.9	12.9	24.4	35.8
	Market wage	12.9	24.3	35.8	12.9	24.2	35.5	12.9	24.4	35.8
Mothers	Marginal effect of w	33.5	39.3	44.3	0.0	0.0	0.0	35.6	40.3	44.9
	Marginal effect of y	-26.3	-24.8	-23.1	0.0	0.0	0.0	-26.5	-25.1	-23.7
	Price of time	8.8	20.6	32.7	21.9	36.5	51.9	7.8	19.3	30.7
	Market wage	7.7	19.2	30.6	7.1	18.4	29.6	7.8	19.3	30.7

Notes: Quartiles are computed using 200 simulated error terms ν and ϵ per observation generated from Halton sequences, resulting in 1270×200 simulated observations. Marginal effects are defined as the impact of a one-dollar change in the hourly wage and a one-thousand-dollar change in annual net total expenditures on annual hours of work.

rejects the linear specification in favor of the quadratic alternative, which includes the squared wage term. Overall, these findings suggest that the quadratic specification provides a more appropriate characterization of the wage–hours relationship than the linear specification.

Interestingly, the differences in marginal effects between mothers and childless women, observed for both wages and net total expenditures in Table 4, highlight the role of children in shaping labor supply responses. In particular, the presence of children reduces the effect of net total expenditures while amplifying the effect of wages, as predicted by the theory and, to some extent, by the empirical evidence discussed above.

Finally, the estimated effects of socio-demographic variables are broadly consistent across specifications and with the existing literature. The first-stage residual is not statistically significant, providing no evidence of endogeneity in net total expenditures.

The middle panel of Table 3 reports the estimated parameters of the child cost equation. As discussed above, the intercept aggregates the constants of the time and monetary cost functions, which cannot be identified separately. Overall, the estimated coefficients have the expected signs, with the exception of the coefficient of net total expenditures in the linear specification, which is negative but not statistically significant. Given the reservations expressed earlier

Table 5: Quartiles of the distribution of total cost of children, with its disaggregation, computed with the quadratic specification and the full sample (QUA)

	All mothers			Non-w. mothers			Working mothers		
	25%	50%	75%	25%	50%	75%	25%	50%	75%
Total cost of one child	15.7	21.5	27.6	23.5	31.0	38.6	15.3	21.0	26.8
Total cost of two children	21.9	30.4	39.2	33.1	43.3	54.0	21.0	29.1	37.2
Total cost of children (aggregate)	18.3	25.7	34.3	28.4	38.3	49.5	17.6	24.7	32.6
<i>Value of mother's time</i>	5.2	12.2	19.8	13.4	22.6	32.7	4.6	11.4	18.4
<i>Value of goods and services</i>	11.4	14.2	16.4	13.1	17.0	18.0	11.3	14.0	16.0

Notes: Quartiles are computed using 200 simulated error terms ν and ϵ per observation generated from Halton sequences, resulting in 1270×200 simulated observations. All costs are in thousands of dollars. Recall that the value of mother's time is given by $wt(w, n)$ and the value of purchased goods and services by $e_m(y, w, n) - wt(w, n)$.

regarding the linear specification, we focus on the three quadratic specifications, which all yield broadly similar results. For the full sample, the estimate effect of net total expenditures amounts to 0.089, indicating that, in a one-child household, the child receives approximately \$89 for every \$1,000 increase in net total expenditures. For the restricted sample and the sample with working women only, the corresponding estimates are 0.174 and 0.209, respectively, implying the child receives approximately \$174 and \$209. In a two-child household, the total amount allocated to both children together increases to about $\$126 \approx \$89 \times \sqrt{2}$ in the full sample, $\$246 \approx \$174 \times \sqrt{2}$ in the restricted sample, and $\$296 \approx \$209 \times \sqrt{2}$ in the sample with working women only. In addition, the estimated effect of the hourly wage is equal to 0.497 for the full sample and 0.190 for the restricted sample, implying that, on average, the mother of a single child devotes approximately 497 and 190 hours per year to childcare. Remarkably, these magnitudes are broadly consistent with external evidence from the 2019 American Time Use Survey (ATUS), which indicates that a comparable single mother with similar characteristics spends about 64 minutes per day on childcare, corresponding to approximately 389 hours per year.¹⁶ The other estimates obtained from the sample of working women only are considerably less precise, exhibiting substantially larger standard errors, and can therefore be safely disregarded for interpretation.

The quartiles of the distribution of the price of time and the market wage obtained with the

¹⁶The ATUS sample used for this comparison is drawn from IPUMS ATUS (Flood, Sayer, and Blackman, 2025) and includes all single mothers under age 60 with one or two children.

full sample are also reported on Table 4. The price of time is used to compute the total cost of children. It coincides with the market wage for working women, but exceeds the market wage for non-working women, reflecting the higher opportunity cost of time outside market employment. In the full sample including both women with and without children, the median price of time for a non-working woman is \$46.8, approximately twice the median market wage of \$23.4. The price of time also varies with parental status. Compared with childless women, single mothers who do not work face lower market wages but also exhibit a lower price of time. This pattern — although a little unexpected — is consistent with their higher labor force participation rates, as documented in the descriptive statistics.

Finally, the annual cost of one child or two children, measured in thousands of dollars, obtained from the estimates of the quadratic specification with the full sample, is reported in the top panel of Table 5 as a function of the labor force participation decision. For the subsample of working mothers, the median annual cost of one child is estimated at approximately \$21,000 per year, with values ranging from about \$15,300 at the first quartile to about \$26,800 at the third quartile. While this figure may appear large at first glance, it is consistent with the fact that the measure incorporates the opportunity cost of parental time, a component typically omitted from alternative estimates of child-rearing costs. By comparison, for the subsample of non-working mothers, the median cost of children is substantially higher, reaching \$31,000 per year, with values ranging from about \$23,500 at the first quartile to about \$38,600 at the third quartile. This difference is driven primarily by a higher implicit price of time, which increases the valuation of time-intensive child-rearing activities when market work is forgone.¹⁷ The cost of two children, also reported in Table 5, is obtained by mechanically rescaling the cost of a single child by a factor of $\sqrt{2}$.

The annual cost of children for all mothers, together with its decomposition, is reported in the bottom panel of Table 5. At the median of the distribution, for working and non-working mothers, the value of a mother’s time and the value of purchased goods and services amount to approximately \$12,200 and \$14,200, respectively. In particular, the latter component can be

¹⁷The use of a higher price of time for non-working women in the computation of the total cost of children is open to debate. For the sake of simplicity and internal consistency, we adopt this convention throughout the analysis. A more thorough conceptual discussion of the appropriate valuation of time for non-working women, however, is beyond the scope of the present paper and is left for future research.

compared to the estimates reported by Lino et al. (2017) for the U.S Department of Agriculture who find annual expenditures of \$11,450–\$13,280 for a single parent with one child and to \$19,210–\$20,660 for a single parent with two children.

5 Conclusion

The central contribution of this paper is the development of a simple and tractable method for incorporating children into labor supply models. The proposed approach is conceptually comparable to equivalence scales, but is specifically designed to be consistent with the theoretical underpinnings of labor supply analysis. Within this framework, children affect parents' labor supply decisions through both income and price effects. This perspective highlights that the common practice of including the number of children merely as an additional control variable in labor supply regressions is insufficient to capture the economic channels through which children influence parental behavior.

Following the typology proposed by Browning (1992), the primary objective of this paper is to address the positive question, namely how the presence of children affects parents' behavior, and in particular their labor supply decisions.¹⁸ Despite this deliberately modest premise, our approach allows us to estimate the total cost of children, including its time component, without relying on direct data about time use or disaggregated consumption. Therefore, using a sample of single women (with and without children) drawn from the PSID, we estimate a labor supply model and show that the presence of children increases the sensitivity of labor supply to wages and reduces its sensitivity to exogenous incomes. The results from our benchmark specification also imply that, on average, single mothers dedicate 497 hours annually to their children. By combining these estimates with an evaluation of the mother's price of time, we are able to recover the total cost of children. For a working mother, for whom the price of time is proxied by the market wage, the estimated total cost of a single child is around \$21,000 per year. For non-working mothers, for whom the price of time is endogenously determined, the estimated cost of a single child increases to approximately \$31,000 per year. Overall, these estimates are broadly consistent with the existing literature and support the robustness of our approach.

¹⁸Browning (1992) distinguishes four interpretations of equivalence scales, corresponding to four distinct questions: the positive question, the needs question, the expenditure question, and the iso-welfare question.

One final remark is warranted. The paper largely, though not entirely, sets aside the question of how to define the price of time, particularly for women who do not participate in the labor market.¹⁹ Using the (reservation) wage rate as the price of time, the cost of children should be interpreted primarily as a cost *perceived by parents*, reflecting forgone consumption of goods and services as well as lost leisure. Accordingly, it does not *necessarily* represent an amount that ought to be compensated to parents on ethical or normative grounds. Addressing these issues may require the construction of an alternative measure of the price of time — one that would be defined in a way consistent with the particular objectives of the study, as the appropriate valuation of time may differ depending on whether the focus is descriptive, normative, or policy-oriented — an issue that lies beyond the scope of the present analysis.

Appendix

Extension to a Couple with Children

The previous model primarily applies to a single woman with and without children, but it can be adapted to represent the behavior of a married woman as well. In the unitary approach, the utility function of the couple is given by:

$$u(h_f + t_f, h_m + t_m, c) + \lambda(n)u_c(m),$$

where h_f , h_m denote the wife's and husband's labor time and t_f , t_m denote the wife's and husband's childcare time.²⁰ The childcare technology is then generalized to:

$$f(t_f, t_m, n) + k \geq T_c(n). \quad (27)$$

From this, it can easily be shown that the wife's working time function becomes:

$$h_f(y, w_f, w_m, n) = F(w_f, w_m, y - e_t(w_f, w_m, n) - e_m(y, w_f, w_m, n)) - t(w_f, w_m, n), \quad (28)$$

¹⁹Even for working women, the price of childcare time does not necessarily coincide with the wage rate, as discussed by Donni and Vil (2025).

²⁰Here, consumption c represents the combined consumption of both spouses. This aggregation is not restrictive and can be justified by the Hicks aggregation theorem.

where w_f, w_m are the wife’s and husband’s wage rate, respectively. In this generalized model, all previously derived results remain applicable. Specifically, the total cost of children, which is supported by the couple as a whole, can be identified from the observation of the sole wife’s labor supply function if the baseline labor supply function is nonlinear. A simplified technology can also be proposed to obtain a more tractable model.

The unitary approach offers the advantage of simplicity, particularly by focusing on the total cost of children. Its main limitation by comparison with the collective approach is that it does not allow for the identification of how the total cost is distributed between the parents. In the absence of distribution factors, however, the unitary approach is only marginally more restrictive than the collective approach. Its key requirement is that the underlying labor supply functions satisfy the Slutsky conditions, which, in the case of a single labor supply equation, essentially reduce to a simple positivity condition.

If the husband’s working time is fixed at full-time — a not entirely unreasonable assumption (Donni, 2007; Donni and Moreau, 2007) — the labor supply function of a married woman has the same structure as in (7), where exogenous income is the sum of the man’s income and non-labor income. While this approach is common when estimating female labor supply functions, it represents a challenge in our context: if the husband’s working time is constrained, his wage rate no longer represents an accurate measure of the price of his time spent on childcare. Identification is therefore no longer guaranteed, but this issue is beyond the scope of the present paper.

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