

The Household Demand for Leisure, the Price of Time and the Full Cost of Children: A Structural Model and Evidence from the PSID*

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Abstract

We propose a novel approach to estimating the full cost of children — the sum of monetary and time costs — by endogenizing the price of parental time rather than assuming it is equal to the parents’ wage rate. In this approach, the price of time depends on how parents perceive their time with children, whether as a leisure-like activity or more as a labor-like activity. We then develop a simplified collective model of leisure demand for working couples, incorporating individual preferences and childcare technology, and estimate it using 2019 PSID data. This allows us to recover the price of parental time and the full cost of children. We find that mothers perceive 44% of their childcare time as labor, compared to 35% for fathers. Our results also highlight that a substantial portion of the full cost of children is non-monetary.

JEL Classification: I30, I31, J21, J22, J31

Keywords: Labor Supply, Leisure Demand, Collective Model, Price of time, Identification, Resource sharing, Cost of children.

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1 Introduction

The cost of children is an important parameter in the analysis of economic policy and inequality. Economists have long developed methods to infer from survey data what parents spend for children (see Bargain and Donni, 2012; Bargain, Donni and Hentati, 2022; Bargain, Lacroix, and Tiberti, 2022; Brown, Calvi, and Penglase, 2021; Calvi et al., 2023; Dunbar, Lewbel and Pendakur, 2013; Lechene, Pendakur, and Wolf, 2022; Lewbel and Pendakur, 2024; Penglase, 2021, for recent applications). Expenditures, however, capture only part of the cost of children. The full cost of children indeed consists of both a monetary cost — the purchase of goods and services that contribute to children’s well-being — and a time cost, which reflects the value of time parents and other caregivers dedicate to children.¹

Evaluating the time cost is particularly complex and has been largely overlooked, with only a few exceptions. Gustafsson and Kjulin (1994) value parents’ time by using either wage rates or the price of equivalent services and then combine this with measures of monetary expenditures to construct a measure of the full cost of children. Apps and Rees (2001) develop a structural model to estimate the full cost of children, incorporating both monetary and time costs. Their model assumes linear homogeneity in childcare technology, identical parental preferences, and that the price of time is determined by the parents’ wage rates. Similarly, Colombino (2000) constructs a simple structural model where the number of children is endogenous and parental time is also valued at the wage rate. Finally, Bradbury (2008) presents a theoretical framework and numerical illustrations, where the full cost of children is inferred from variations in parents’ leisure, following a Rothbarth-style approach.²

One difficulty in measuring the time cost is to determine how parental childcare time should be valued. Childcare combines a productive dimension with a consumption dimension, and these

¹To be comprehensive, other long term costs may also arise, such as reduced career advancement opportunities (see Adda, Dustmann, and Stevens, 2017; Budig and England 2001; Dechter, 2014; Glauber, 2018; Korenman and Neumark, 1992; Waldfogel, 1998).

²Koulovatianos, Schröder and Schmidt (2009) adopt a different identification strategy, recovering the full cost of children from subjective questions rather than observed parental behavior.

two aspects are inherently linked in determining its value. Reid’s (1934) third-party criterion provides a natural starting point for characterizing the productive component: an activity is productive when it is undertaken to generate an outcome that could, in principle, be provided by a third party. At the same time, childcare may generate direct utility for parents beyond its productive outcome, as captured by the concept of process benefits introduced by Juster and Stafford (1991). The relevant object for measuring the time cost is therefore the price of the productive component of childcare time. Typically, this price is assumed to equal the parents’ wage rate. This approach, however, implicitly relies on two strong conditions: (a) the spouse must participate in the labor market and be able to adjust working time sufficiently freely for the observed wage to provide a meaningful measure of the price of time; and (b) the time devoted to the activity must be purely productive (i.e., without process benefits), so that it is perfectly substitutable at the margin for market work, with a marginal rate of substitution equal to one. The first condition is a general difficulty in valuing household production and depends largely on labor market institutions and constraints. It will not be addressed in this paper. The second is particularly problematic for childcare. Consistent with the notion of process benefits, Cosaert and Hennebel (2023) indeed show that a large fraction of childcare time is perceived by parents as leisure. These considerations suggest that the price of parental childcare time should be determined endogenously from parental preferences and the childcare technology, rather than simply equated to the market wage.

The aim of this paper is to evaluate the full cost associated with raising children, using a structural framework that identifies the productive component of parental childcare time and defines the corresponding price of parental childcare time. We develop a collective labor supply model for a working couple with children. Each parent has a distinct utility function that depends on his or her leisure and childcare time, own consumption, and children’s consumption, with the crucial feature that childcare time is not necessarily equivalent to market work from the parent’s perspective. More generally, childcare time can be viewed by parents as either a leisure-like activity that increases utility — reflecting positive process benefits — or a labor-like activity that has no direct effect on utility or may even reduce it, corresponding to negative process benefits.³

³Market work does not enter utility directly and therefore provides the natural benchmark. Childcare with

In addition to influencing utility, childcare time also enters a childcare technology together with purchased childcare services. Inspired by Reid’s (1934) third-party criterion, this technology does not represent the production of childcare itself, but rather the substitution possibilities between parental childcare time and market childcare services. The price of parental childcare time is determined by the marginal rate of substitution between these two inputs, while its productive value reflects the extent to which it substitutes for market childcare. To complete the model, we finally assume that parents make expenses for their children according to a predetermined rule. The full cost of children is finally assumed to comprise both the value of parental time devoted to the productive childcare activity and these direct expenses for children.

Our main result is that the full cost of children can be identified from observed behavior. Specifically, our model allows us to break down each unit of childcare time into a productive childcare activity and a leisure-like (or labor-like) activity that directly affects the parents’ utility. We then estimate this model using a sample of dual-earner couples, both with and without children, from the 2019 wave of the Panel Study of Income Dynamics (PSID). The data provide information on time use, expenditures including external childcare, and a range of socio-demographic characteristics. The estimation has two stages. First, we estimate the childcare technology and determine the price of productive childcare time, enabling us to calculate the time cost of children and define a composite measure of full leisure that includes the leisure-like (or labor-like) component of childcare activities. Second, we estimate full leisure demand functions to derive the full cost of raising children in a Rothbarth-like approach. Our estimates indicate that, depending on the specification, 32–66% of mothers’ childcare tasks are effectively productive childcare time, compared with 20–42% for fathers, implying the price of productive childcare time is substantially below market wages. For a dual-earner couple with one child, the time cost of raising children is estimated at approximately \$230–\$570 per week at the median of the sample while the monetary cost at approximately \$430–\$890 per week. However, the monetary cost estimates are more fragile, for reasons discussed below.

no direct utility effect is equivalent to market work, while positive (negative) process benefits make it more leisure-like (labor-like). The work–leisure distinction is therefore continuous rather than binary.

To the best of our knowledge, this is the first study to use endogenously determined shadow prices to estimate the time cost of children. This contrasts with existing approaches, which either value time at market wages or focus exclusively on consumption expenditures, thereby capturing only the monetary component of the cost of children. Our procedure then recovers the full cost of children (the monetary cost is obtained residually as the difference between the full cost and the time cost). While somewhat similar to Rothbarth-like approaches — which compare adult consumption between couples with and without children — our method to recover the full cost of children introduces several key innovations. The ‘adult good’ we analyze is parents’ full leisure, which we define to include a portion of childcare time, with this fraction determined by a childcare technology function. Moreover, the demand equations are conditioned on full expenditure, i.e., the sum of household total expenditure on goods and services and expenditure in leisure time computed with spouses’ wage rates as prices, rather than on conventional total expenditure. Our approach thus differs from approaches which focus on consumption goods rather than time use.⁴ Moreover, unlike Bargain and Donni (2012) and Bargain, Donni, and Hentati (2022), we do not rely on data from single individuals and, unlike Dunbar, Lewbel, and Pendakur (2013), we refrain from imposing strong parametric restrictions on inter-individual preferences, while retaining their Similarity Across Types (SAT) condition. Conversely, our contribution can be related to the few studies that have attempted to estimate process benefits, though not specifically in the context of childcare. Graham and Green (1984) were pioneers in this area, extending Gronau’s (1977) classical domestic production model while maintaining perfect substitutability between market goods and household production, but they did not address identification issues. Kerkhofs and Kooreman (2003), using a closely related framework, provide a thorough analysis of the identification of process benefits and show that identification can be achieved under certain configurations; Gørtz (2011) further extends this analysis. These studies, however, do not consider the additional complexities introduced by the presence of children. Cosaert and Hennebel (2023) are closer to our analysis because they explicitly consider process benefits in the context of childcare. They show that process benefits are set-identified rather

⁴Our approach also differs from that of Arduini (2024), who relies on measures of leisure time but does not focus on children’s cost.

than point-identified, but their revealed-preference approach is fundamentally different from ours. Moreover, their objective is not to estimate the full cost of children or either of its time and monetary components.⁵

The rest of this paper is as follows. Section 2 presents the theoretical model. Section 3 outlines the empirical specification and the data. Section 4 describes the estimation method and the empirical results. Finally, Section 5 concludes.

2 The Model

2.1 Decision-making in a couple with children

We consider a two-adult household, consisting of a wife (W) and a husband (H), with n children, who make decisions about leisure, childcare, and consumption in a static framework.⁶ The wife's and husband's leisure time and consumption are respectively denoted by l_W , l_H , c_W and c_H while their childcare time are respectively denoted by t_W and t_H and the children's consumption by c_K . Spouses have specific preferences for how they allocate their time and consumption. More precisely, each spouse I , with $I = W, H$, has a utility function of the form:

$$U_I = U_I(l_I, t_I, c_I, c_K, n),$$

where U_I is a differentiable function, strictly increasing in l_I , c_I , and c_K , and strongly concave in l_I , t_I , c_I and c_K . This specification generalizes the utility function used by Blundell, Chiappori and Meghir (2005).⁷ It allows for general interactions between parental childcare time and

⁵Hallberg and Klevmarken (2003) use direct survey questions to elicit the process benefits associated with time spent on childcare.

⁶The terms 'wife' and 'husband' are used here for simplicity, and the partners are not necessarily married. As for the variable n , it may more generally be interpreted as a vector that also includes child characteristics such as age and sex.

⁷For simplicity, the baseline model abstracts from domestic production beyond childcare, public consumption, externalities, mutual altruism, and any kind of interactions between spouses' utility functions. We show,

consumption. In particular, utility is increasing in children's consumption, suggesting that parents are altruistic toward their children. Parental childcare time may also enter utility. This may reflect process benefits in the traditional sense, whereby parents derive direct utility or disutility from performing the childcare activity itself. More generally, because parents may also value the effects of childcare on their children's well-being, childcare time may affect parental utility through this additional channel. At this stage, we see no compelling reason to restrict how childcare time affects utility. In what follows, the term process benefits is thus used in a broader sense than usual to encompass both effects. When the process benefits associated with childcare are positive, $\partial U_I / \partial t_I > 0$, and childcare is leisure-like at the margin. When they are non-positive, $\partial U_I / \partial t_I \leq 0$, and childcare is labor-like at the margin.⁸ Finally, we assume that $\lim_{c_I \rightarrow 0} \partial U_I / \partial c_I = \lim_{c_K \rightarrow 0} \partial U_I / \partial c_K = +\infty$, which ensures that consumption is always strictly positive in equilibrium; and $\lim_{l_I \rightarrow 0} \partial U_I / \partial l_I = +\infty$ with $\partial U_I / \partial t_I < \partial U_I / \partial l_I$. This latter restriction ensures that the implicit price of parental childcare time is positive, as required by the childcare cost minimization problem introduced below. Together with $\lim_{l_I \rightarrow 0} \partial U_I / \partial l_I = +\infty$, this condition ensures that leisure time is always strictly positive in equilibrium.

For a given number of children n , we assume that parents must meet some given childcare requirement. This requirement can be satisfied through parental childcare time, externally purchased childcare services, or a combination of the two. In the linear case, parental and purchased childcare are perfect substitutes in meeting this requirement: an additional hour of parental childcare then displaces a fixed amount of purchased childcare. Parental childcare has then a clear productive dimension in the sense of Reid's (1934) third-party criterion, since it provides a service that can alternatively be supplied by a third party, which is distinct from the utility parents may derive from performing childcare itself.⁹ In the general case, the replacement rate

however, in Section 2.2.4 that the tractable specification used below remains compatible with these features under additional assumptions.

⁸More precisely, childcare time is locally equivalent to market work when $\partial U_I / \partial t_I = 0$. When $\partial U_I / \partial t_I < 0$, childcare is more unpleasant than market work, since it reduces utility directly in addition to displacing leisure.

⁹If parental childcare also affects parents' utility through its effects on children's well-being, the childcare technology captures only the component of parental childcare that substitutes for externally provided care. The

may vary with the allocation of parental childcare time, in particular, because the amount of purchased childcare displaced by an additional hour of parental care may depend on how the parents can coordinate with each other. For a given number of children n , let $g(t_W, t_H, n)$ denote the minimum expenditure on external childcare services required for given amounts of parental childcare time. The childcare requirement is represented by

$$m - g(t_W, t_H, n) \geq 0, \quad (1)$$

where m is the money dedicated to purchase external childcare services. We assume that g is differentiable, strictly decreasing and strictly convex in its first two arguments — which guarantees that isoquants between m and t_W or t_H are decreasing and convex with respect to the origin — and increasing in its last argument. Additionally, we assume that

$$m \geq 0, \quad (2)$$

which ensures that external childcare expenditures can be zero or positive but never negative. For simplicity, we rule out the possibility of zero childcare time, by assuming $\lim_{t_W \rightarrow 0} \partial g / \partial t_W = \lim_{t_H \rightarrow 0} \partial g / \partial t_H = -\infty$.

A few remarks are in order. First, rather than introducing a separate childcare output and modeling its endogenous choice by parents, we adopt a more parsimonious approach that directly characterizes substitution between parental time and purchased childcare. In a more general model with an explicit childcare output entering parental utility functions, this substitution would generally depend on the endogenously chosen level of the childcare output and therefore indirectly on household resources and input prices. Our conditional representation is thus more restrictive, but also simpler. Second, the baseline formulation does not distinguish childcare provided jointly by both parents from childcare provided by either parent alone. It therefore abstracts from any efficiency gains or losses associated specifically with joint care. This restric-

estimated time cost can then be interpreted as the cost of parental childcare conditional on a given childcare requirement, excluding any additional benefits of parental care for children's well-being that are valued by parents.

tion is not essential, however, as joint childcare can be incorporated into the technology under fairly mild conditions.¹⁰ Third, childcare time, t_W and t_H , is defined strictly as the time actively dedicated to caring for children, that is, time that is explicitly taken away from either paid work or leisure. This definition excludes passive supervision, such as merely being present while performing other activities, because such time is assumed to entail a negligible opportunity cost for parents.¹¹ Under this assumption, the household traditional budget constraint — with the prices of both parents' and children's consumption goods normalized to one — is of the form:

$$Y - c_W - c_H - c_K - l_W w_W - l_H w_H - t_W w_W - t_H w_H - m \geq 0 \quad (3)$$

where $Y = T \cdot (w_W + w_H) + y$ is the household full income, T the total time endowment of each spouse, y other nonlabor income, and w_W and w_H the spouses' wage rates.

The decision process is assumed to lead to Pareto efficient outcomes. For a couple with children, the optimization problem of the household is thus:

$$\max_{l_W, l_H, t_W, t_H, c_W, c_H, c_K, m} \phi_W U_W(l_W, t_W, c_W, c_K, n) + \phi_H U_H(l_H, t_H, c_H, c_K, n) \quad (\bar{P})$$

with $\phi_W + \phi_H = 1$, subject to the childcare constraint (1), the non-negativity constraint (2), the budget constraint (3) and time feasibility conditions: $T - l_W - t_W \geq 0$ and $T - l_H - t_H \geq 0$. In

¹⁰If joint childcare can be expressed as a function of the parents' total childcare times, the function g can be interpreted as the reduced form of a more general technology g^* distinguishing joint from individual childcare:

$$g(t_H, t_W, n) = g^*(t_H - J(t_H, t_W, m, n), t_W - J(t_H, t_W, m, n), J(t_H, t_W, m, n), n)$$

where $j = J(t_H, t_W, m, n)$ denote the amount of childcare jointly provided by both parents, with $0 \leq J(t_H, t_W, m, n) \leq \min(t_H, t_W)$. This assumption is unlikely to be restrictive insofar as parents' total childcare times summarize the effects of the exogenous variables determining the allocation of childcare.

¹¹As recently emphasized by Cuevas-Ruiz et alii (2025), passive supervision may still impose costs by restricting parents' location, availability, or simultaneous activities, or by reducing the quality of leisure and productivity of other tasks. We abstract from these costs and assume they are small relative to those of active childcare. As discussed below, however, the PSID uses a broader measure of childcare that includes passive supervision, complicating the empirical implementation of our framework.

this problem, ϕ_W and ϕ_H are Pareto weights that may generally depend on all the exogenous variables. They determine the location along the Pareto frontier. If $\phi_W = 0$ (or $\phi_H = 1$), then the household behaves as though the husband always gets his way, whereas, if $\phi_W = 1$ (or $\phi_H = 0$), it is as if the wife is the effective dictator. If Pareto weights are constant, the optimization problem simplifies to the maximization of a separable household utility function, consistent with the unitary approach. If instead Pareto weights depend on wage rates, other exogenous incomes, or distribution factors — i.e., variables that affect bargaining power without entering the budget constraint — the optimization problem corresponds to the collective approach *stricto sensu*. Finally, the expenditure allocated to the child reflects the interaction between parents' altruism and their relative bargaining power.

As is common with collective models (see Donni and Chiappori, 2011), the optimization problem can be decentralized. If we assume that the time feasibility conditions are not binding at the optimum (implying that market work time is strictly positive), we have the following result.

Proposition 1. *Assume that time feasibility conditions are not binding. Then the optimal allocation of time and consumption in Problem $\bar{P}$ can be represented as the solution of a decentralized decision process. More precisely, there exists a unique pair of positive functions (θ_W, θ_H) defined on (w_W, w_H, Y, n) such that:*

(a) *the time cost is minimized:*

$$e_T(w_W, w_H, Y, n) = \min_{t_W, t_H} (t_W \theta_W w_W + t_H \theta_H w_H + g(t_W, t_H, n)) \quad (P_0)$$

subject to $g(t_W, t_H, n) \geq 0$

(b) *the household utility is maximized:*

$$\max_{\kappa_H, \kappa_W, c_K} \{ \phi_W V_W(\kappa_W, c_K; w_W, \theta_W, n) + \phi_H V_H(\kappa_H, c_K; w_H, \theta_H, n) \} \quad (P_1)$$

subject to $\kappa_W + \kappa_H + c_K \leq Y - e_T$, where

$$V_I(\kappa_I, c_K; w_I, \theta_I, n) = \max_{l_I, t_I, c_I} U_I(l_I, t_I, c_I, c_K, n) \quad (\text{P}_I)$$

subject to $\kappa_I - c_I - l_I w_I - t_I(1 - \theta_I)w_I \geq 0$, for $I = H, W$.

The proofs are in Appendix A. Focusing on interior solutions for market work time is important because it ensures that wage rates are observed and represent a convenient measure of the price of time.¹² Intuitively, the spouses first minimize the time cost required to satisfy the childcare constraint, with $\theta_I w_I$ as the price of the productive component of parental childcare time (Problem P_0). They then allocate their remaining resources between themselves and their children's consumption according to the functions κ_W , κ_H and c_K in order to maximize the household utility function (Problem P_1), using conditional indirect utility functions V_I obtained from individual optimization problems (Problems P_H and P_W). In the individual problems, childcare time is valued at $(1 - \theta_I)w_I$. Each unit of childcare time by spouse I is therefore decomposed into a productive childcare component valued at $\theta_I w_I$ and a component that directly affects parental utility, valued at $(1 - \theta_I)w_I$. The latter is a leisure-like activity when $\theta_I < 1$ and a labor-like activity when $\theta_I \geq 1$. If $\theta_I = 1$, there is no process benefits; similar to market labor time, childcare time simply reduces parents' leisure time.¹³ Taken together, the shadow prices $w_I^* = \theta_I w_I$ and $\bar{w}_I^* = (1 - \theta_I)w_I$ can be interpreted as Lindahl prices, similar to those encountered in public economics,¹⁴ and satisfy $w_I^* + \bar{w}_I^* = w_I$.

The time cost of children, defined as $e_T(w_W, w_H, Y, n)$, is evaluated using the prices $w_W^* = \theta_W w_W$ and $w_H^* = \theta_H w_H$, rather than the standard wage rates. If childcare is leisure-like, so that

¹²As explained in the proof, however, this result can be extended to the case in which the time feasibility conditions are binding. Observed wage rates are then replaced by shadow wage rates, and full income is adjusted accordingly. Although the decentralized representation remains unchanged, the shadow wages are not directly observed and must be recovered, which complicates identification of the prices of time.

¹³The case where $\theta_I \leq 0$ is excluded by the assumption $(\partial u_I / \partial t_I) / (\partial u_I / \partial l_I) < 1$. Without this assumption, the analysis would be significantly more complicated.

¹⁴See also Donni (2007, 2009) for an application to the household.

$w_I^* < w_I$, valuing parental time at market wages overstates the time cost of children, *ceteris paribus*. Conversely, if childcare is labor-like, so that $w_I^* \geq w_I$, the valuation based on market wages is exact when $w_I^* = w_I$ and understates the time cost when $w_I^* > w_I$. The full cost is defined as the sum of the time cost $e_T(w_W, w_H, Y, n)$ and the monetary cost $e_M(w_W, w_H, Y, n)$, the latter being equal to $c_K(w_W, w_H, Y, n)$.

Finally, it is important to note that this result continues to hold when the constraint (2) is binding, i.e., when the household does not purchase external childcare services. In that case, the Kuhn–Tucker multiplier associated with this constraint may be positive, giving rise to an adjustment factor $\nu \geq 1$, with $\nu = 1$ whenever $m > 0$.¹⁵ This factor affects only the marginal conditions determining the optimal mix of parental childcare times when $m = 0$; it does not affect the parents' utility maximization problems. The same adjustment factor applies to both spouses, and the marginal reduction in the amount of external childcare required by an additional unit of parental childcare is therefore $\nu\theta_I w_I = -\partial g / \partial t_I$.

2.2 Identification and Empirical Tractability

The primary objective of this study is to show how the full cost of children — and its decomposition into time cost and monetary cost — can be identified from traditional survey data. The childcare technology, and hence the time cost of children, can be identified under fairly general conditions. Identifying the remaining components of the model requires additional restrictions that will be detailed below.

2.2.1 Identification of the Time Cost of Children

As is usual in the literature on the identification of collective models, we first assume that individual consumptions c_W , c_H and c_K are not observed by the economist. Instead, only their sum $c_W + c_H + c_K$ is observable. The other dependent variables, l_W, t_W, l_H, t_H and m , are observed as functions of the exogenous variables. The optimal solutions of the cost minimization problem are denoted by m^* , t_W^* and t_H^* each expressed as a function of (w_W, w_H, Y, n) .

¹⁵The details are given in the proof of Proposition 1 in Appendix A.

The first identification result is the following.

Proposition 2. *Assume that time feasibility conditions are not binding. Under regularity and support conditions, the childcare technology can be recovered from the observation of m^* , t_W^* , and t_H^* as functions of (w_W, w_H, Y, n) . The scalar θ_W and θ_H are exactly identified when $m^* > 0$, and are identified up to a common multiplicative function when $m^* = 0$.*

Intuitively, when $m^* > 0$, the prices of productive childcare time, $w_H^* = \theta_H w_H$ and $w_W^* = \theta_W w_W$, correspond to the marginal rate of technical substitution between t_W and t_H on the one hand, and m on the other. From these marginal rates of substitution, the corresponding price-scale coefficients θ_H and θ_W are then directly recovered. When $m^* = 0$, the non-negativity constraint on m is binding, so that these marginal rates identify νw_H^* and νw_W^* , with a common factor $\nu \geq 1$. Hence, only the ratio of the price-scale coefficients is identified nonparametrically. Full identification requires further restrictions that will be discussed below.

Identifying the other component of the full cost of children raises additional difficulties. The general principle underlying almost all existing approaches is to compare the behavior of couples with children with that of otherwise similar couples without children. More formally, these approaches rely on the existence of one or more stable relationships among at least two observable variables that are invariant to the presence of children once the resources devoted to them have been accounted for, and that can be recovered from observed behavior. The full cost of children is then identified as the resource adjustment required to restore these relationships when comparing otherwise similar couples with and without children. To derive these stable relationships, we proceed in two stages, developed in the next subsections.

2.2.2 The Full Leisure Demand Functions

The decision process described in Proposition 1 is not strictly speaking sequential as the first-stage Problem P_0 depends on the prices of productive childcare time, which are also determined by the second-stage Problems P_1 . To obtain a clear sequential decision framework, we adopt a classical assumption used in all models that incorporate process benefits (Gørtz, 2011; Gra-

ham and Green, 1984; Kerkhofs and Kooreman, 2003; Cosaert and Hennebel, 2023; Donni and Matteazzi, 2018), imposing additional structure on the utility functions as described below.

Assumption A.1 Each spouse I has a utility function of the form:

$$U_I = U_I(l_I + \varphi_I(t_I), c_I, c_K, n)$$

with $I = W$ or H , where $L_I = l_I + \varphi_I(t_I)$ can be viewed as the ‘full leisure’ of spouse I for some differentiable and concave functions $\varphi_I(\cdot)$, satisfying $\varphi_I(0) = 0$.

This additional structure preserves the core properties of our model while making it more tractable for empirical estimation. It offers several advantages. Firstly, leisure time and childcare time are combined into a single composite full leisure time. This definition can be applied consistently to both couples with and without children and facilitates comparisons between them. Secondly, the decision process is a sequential one. Specifically, the price of the leisure-like (or labor-like) activity in the second-stage problem is determined by the first-stage choice of childcare time only, as detailed in the following proposition. Thirdly, the assumption provides the additional structure needed to fully identify the price-scale coefficients: it resolves the multiplicative indeterminacy that otherwise arises when external childcare expenditure is zero.

The following proposition first establishes the sequential structure of the decision process.

Proposition 3. *Assume that time feasibility conditions are not binding. Then, under Assumption A.1, the optimal allocation of time and consumption in Problem $\bar{P}$ is sequential. Firstly, childcare time is determined by the Problem P_0 with $\theta_W = 1 - \varphi'_W(t_W)$ and $\theta_H = 1 - \varphi'_H(t_H)$, giving $t_H^*(w_W, w_H, n)$ and $t_W^*(w_W, w_H, n)$ as optimal levels of childcare time. In particular, both optimal childcare times and the associated implicit price parameters are independent of nonlabor income Y . Secondly, the remaining resources are allocated by the Problem P_1 , where each spouse’s indirect utility is generated by the corresponding Problem P_I , giving κ_H , κ_W , and c_K . In Problem P_I , spouse I maximizes her or his own utility function $U_I(L_I, c_I, c_K, n)$ with respect to (L_I, c_I) subject to the budget constraint $\kappa_I + \pi_I - c_I - L_I w_I \geq 0$, giving $L_I^*(w_I, \kappa_I + \pi_I, c_K, n)$ as optimal levels of full leisure, with $\pi_I = \varphi_I(t_I^*)w_I - t_I^*(1 - \theta_I)w_I$ and $\theta_I = 1 - \varphi'_I(t_I^*)$.*

This result has several useful implications. First, the prices of parental childcare time are determined entirely at the cost minimization stage and, apart from the functions φ_I , independently of the remaining features of individual preferences. Second, the resulting childcare demand functions are independent of nonlabor income Y and, more generally, of variables that affect utility without entering the childcare constraint. This constitutes a strong and testable implication of the model. Once childcare time has been determined, spouse I 's utility $U_I(L_I, c_I, c_K, n)$ is maximized subject to the budget constraint: $c_I + L_I w_I \leq \kappa_I + \pi_I$. In this representation, the function π_I can be viewed as a profit function associated with the production of full leisure: full leisure is valued at the wage rate w_I , while childcare time enters as an input with implicit price $\bar{w}_I^* = (1 - \theta_I)w_I$. In the empirical application, we will adopt an even more simplified formulation in which the functions $\varphi_I(t_I)$ are linear in t_I , so that the functions π_I reduce to zero. This specification also admits a particularly intuitive interpretation: childcare time can be decomposed into a productive domestic-work component and a leisure-equivalent component associated with process benefits. We return to this interpretation in the empirical section.

The additional structure also has useful implications for identification.

Corollary 4. *Under Assumption A.1 and the conditions of Proposition 2, the functions $\theta_I(t_I) = 1 - \varphi'_I(t_I)$, for $I = W, H$, are identified over the entire relevant support of parental childcare time. Consequently, when $m^* = 0$, the multiplier ν is identified from*

$$\nu = -\frac{\partial g / \partial t_I}{\theta_I(t_I^*)w_I}, \quad I = W, H.$$

The idea is that variation in the region where $m^* > 0$ identifies $\theta_I(t_I)$ at all childcare time values that are relevant when $m^* = 0$. Once the functions $\theta_I(t_I)$ are identified over the relevant support, the common multiplier ν can be recovered directly from the first-order conditions. The fact that either spouse yields the same value of ν also provides a restriction of the model.

2.2.3 Identification of the Monetary Cost of Children: General Case

By application of the previous results, the technology g and the price-scale coefficients θ_I are identified. Since $\theta_I = 1 - \varphi'_I(t_I)$, the identified price parameter can be expressed as a function of t_I alone. The function $\varphi_I(t_I)$ can then be recovered by integrating $1 - \theta_I(t_I)$, using the boundary condition $\varphi_I(0) = 0$. Once φ_I is recovered, full leisure time can then be constructed as $L_I = l_I + \varphi_I(t_I)$ and the corresponding profit function π_I can be recovered as well. From now, we assume that optimal solutions L_W^* and L_H^* as well as the utility profit and the time cost are observed as functions of (w_W, w_H, Y, n) .

The remaining structural components of the model, and in particular children's consumption c_K can also be identified. The basic idea is to identify the full cost of children and recover the monetary cost as the residual after the time cost has been identified. This strategy, however, requires additional restrictions. We therefore impose a further assumption stating that some dimensions of parental preferences are invariant to the presence of children.

Assumption A.2. Individual utility functions are separable between adult-related variables and child-related variables, so that

$$U_I = U_I(u_I(L_I, c_I), c_K, n),$$

with $I = W$ or H , where u_I is a differentiable and concave function.

This type of assumption, in one form or another, is standard in the literature. Blundell, Chiappori, and Meghir (2005) impose a very similar separability condition. Gronau (1991), Bargain and Donni (2012), and Bargain, Donni, and Hentati (2022) derive a related separable structure from caring preferences, whereby parents care about their children's utility rather than directly about the goods they consume. Technically, it simply implies that the marginal rate of substitution between L_I and c_I is independent of both the number of children and their consumption. This invariance is crucial for identification, as it provides the stable relationship between the behavior of couples with and without children that is needed to recover children's consumption. However, this assumption may be regarded as excessively strong and it can be relaxed and

replaced, for instance, by the following alternative.

Assumption A.2'. Individual utility functions take the form:

$$U_I = U_I(u_I(L_I, c_I, n), c_K, n),$$

with $I = W$ or H , where u_I is a differentiable and concave function, such that children affect the corresponding demand for full leisure only through an additive demographic term:

$$L_I = \alpha_I(w_I, n) + \beta_I(w_I, \kappa_I).$$

for some functions $\alpha_I(w_I, n)$ and $\beta_I(w_I, \kappa_I)$.

This specification implies that the number of children leaves the slope of the Engel curve unchanged while shifting its intercept — an assumption often referred to as Similarity Across Types (SAT). Although still restrictive, it is weaker than Assumption A.2. The child-specific term $\alpha_I(w_I, n)$ may capture differences in preferences between parents and non-parents, as well as economies of scale associated with raising children.

In what follows, we will use these two assumptions. The remaining difficulty is that individual consumptions c_H and c_W are not separately observed, preventing the direct exploitation of a stable relationship between these variables and full leisure L_H and L_W . From standard results in the literature (Chiappori, 1988, 1992, or Chiappori, Fortin and Lacroix, 2002, for instance), however, individual consumptions as well as the sharing functions κ_W and κ_H can be identified up to a unique constant from a sample of childless couples provided that certain regularity conditions are satisfied. Under the stability condition in Assumption A.2, the full leisure demand functions identified from childless couples also apply to couples with children. Hence,

$$L_I = L_I(w_I, \kappa_I(w_H, w_W, Y_R, n) + k_I),$$

for couples with children, where the functions $L_I(\cdot)$ are identified, while k_W and k_H represent the remaining indeterminacy in the sharing rule and satisfy $k_W + k_H = 0$. The spouses' resource

shares satisfy the adding-up condition:

$$Y + \pi_W + \pi_H - e_M - e_T = \kappa_W + \kappa_H.$$

Provided that each full leisure demand function can be inverted with respect to its second argument, we obtain:

$$\kappa_I + k_I = L_I^{-1}(w_I, L_I).$$

Summing these expressions across spouses eliminates the unidentified constants because $k_W + k_H = 0$, yielding:

$$Y + \pi_W + \pi_H - e_M - e_T = L_W^{-1}(w_W, L_W) + L_H^{-1}(w_H, L_H)$$

Hence, although the individual sharing functions are identified only up to an additive constant, their sum — and therefore the resources remaining after accounting for children — is exactly identified. Since Y , π_W , π_H , and the time cost e_T are known, this identity identifies the monetary cost e_M .

The identification result above relies on the preference restriction embodied in Assumption A.2. An alternative is to relax this assumption and instead exploit nonlinearities in the full leisure demand functions. Specifically, if Assumption A.2 is replaced by Assumption A.2', the inversion of the demand function is replaced by the inversion of its derivative with respect to remaining full income. Formally, let

$$D_I(w_I, \kappa_I + k_I) = \frac{\partial L_I}{\partial(\kappa_I + k_I)}$$

denote the marginal response of full leisure to individual resources. This function is independent of the number of children n . If $D_I(w_I, \cdot)$ is invertible, then

$$\kappa_I + k_I = D_I^{-1}\left(w_I, \frac{\partial L_I}{\partial(\kappa_I + k_I)}\right).$$

The remainder of the proof then proceeds exactly as above. We then present the formal result.

Proposition 5. *Under Assumption A.1, the conditions of Proposition 2 and the regularity conditions ensuring identification of the collective labor supply model (Chiappori, 1988, 1992), the full cost of children is identified from the observation of the full leisure demand functions L_W^* and L_H^* as functions of (w_W, w_H, Y, n) , provided that either: (i) Assumption A.2 holds and L_W and L_H are invertible with respect to individual shares; or (ii) Assumption A.2' holds and the corresponding marginal response functions D_W and D_H are invertible.*

The identifying assumptions generate several testable implications. First, the collective labor supply structure itself imposes the usual restrictions derived by Chiappori (1988, 1992), which can be tested independently of the additional stability assumptions introduced here. Second, the additional stability restrictions are themselves testable through nested specifications. Assumption A.2 implies that full leisure demand is invariant to n , whereas A.2' allows level shifts but requires marginal responses to individual resources to remain invariant. Thus, A.2 can be tested against A.2', and A.2' against a more general alternative.

2.2.4 A More Tractable and Robust Approach

The previous approaches are quite general, but they rely on the regularity conditions required for the identification of a collective labor supply model. In particular, identification requires sufficiently rich nonlinearities in both behavioral relationships.¹⁶ Since our primary objective is to identify the cost of children rather than the exact allocation of full income between spouses, we adopt a more tractable strategy and impose the following assumption.

Assumption A.3 The sharing functions are invariant to the presence of children:

$$\kappa_I(w_H, w_W, Y_R) = \kappa_I(w_H, w_W, Y_R, n),$$

¹⁶As an alternative, identification may rely on the existence of valid distribution factors, which is also a restrictive requirement (Chiappori, Fortin and Lacroix, 2002).

with $I = W, H$, where Y_R denotes remaining full income, defined by $Y_R = Y$ for a couple without children and $Y_R = Y + \pi_W + \pi_H - e_M - e_T$ for a couple with children.

This assumption is automatically satisfied under the unitary approach, in which bargaining weights are constant, but it is more general than that special case. Taken together, Assumptions A.1-A.3 imply that the presence of children affects full leisure only through a pure income effect, without altering individual preferences between L_I and c_I or the intra-household allocation rule between parents. That is, children may modify each parent's share of full income only insofar as it changes Y_R itself. This is the central idea underlying the traditional Rothbarth approach. In addition, unlike the preceding identification result, which relies closely on the collective framework of Chiappori (1988, 1992), the reduced-form representation derived below under Assumption A.3 is robust to a broad range of interactions between spouses' choices that are not explicitly modeled here, including those arising from mutual altruism, public consumption, externalities, or domestic production. For the purpose of identifying the full cost of children, these interactions need not be separately modeled, provided that they do not cause the presence of children to affect behavior independently of remaining full income.

Then we simply define reduced-form full leisure demand functions $F_I(w_H, w_W, Y_R)$, as follows:

$$L_I = L_I(w_I, \kappa_I(w_H, w_W, Y_R)) = F_I(w_H, w_W, Y_R).$$

These functions are invariant to the presence of children; they therefore apply to both childless couples and couples with children and can consequently be identified from a sample of childless couples.¹⁷ For couples with children, remaining full income Y_R is equal to $Y + \pi_W + \pi_H - e_M - e_T$. In this case, we have:

$$L_I = F_I(w_H, w_W, Y + \pi_W + \pi_H - e_M - e_T), \tag{4}$$

¹⁷Under the baseline collective framework, the functions $F_I(\cdot)$ inherit the restrictions derived by Chiappori (1988, 1992) but these restrictions need not hold under the more general interactions discussed above. They are therefore not discussed in what follows.

where $L_I = l_I + \varphi_I(t_I)$. If $F_I(w_H, w_W, \cdot)$ is invertible, the reduced-form demand function can be inverted with respect to the remaining full income, which allows for exact identification of the full cost of children. Specifically, we obtain:

$$e_M = Y + \pi_W + \pi_H - e_T - F_I^{-1}(w_H, w_W, L_I), \quad (5)$$

where all the terms on the right-hand side are either observable or identifiable. Apart from the specific definition of full leisure adopted here, this approach is closely related to that proposed by Bradbury (2008). Identification requires the regularity condition that the Engel curve for full leisure is locally invertible with respect to remaining full income. However, no additional nonlinearity condition is required, which makes the approach considerably simpler to implement than the approach based on the standard collective framework. In addition, the full cost of children can be identified from a single spouse's full leisure demand function.

The same result can be obtained under Assumption A.2'. The reduced-form full leisure demand function is:

$$F_I(w_H, w_W, Y_R, n) = \alpha_I(w_I, n) + \gamma_I(w_H, w_W, Y_R),$$

where $\gamma_I(w_H, w_W, Y_R) = \beta_I(w_I, \kappa_I(w_H, w_W, Y_R))$. Identification then relies on the marginal response of full leisure to remaining full income. Define:

$$D_I(w_H, w_W, Y_R) = \frac{\partial F_I(w_H, w_W, Y_R, n)}{\partial Y_R} = \frac{\partial \gamma_I(w_H, w_W, Y_R)}{\partial Y_R}.$$

Since the additive term $\alpha_I(w_I, n)$ disappears upon differentiation, the function D_I is invariant to the presence of children.¹⁸ If $D_I(w_H, w_W, \cdot)$ is locally invertible, then identification proceeds as before. The drawback here is that identification rests entirely on the curvature of this equation. When curvature is weak, the resulting identification may therefore be fragile.

¹⁸In the empirical specification, we allow the additive demographic term to depend on both spouses' wage rates. This is more general than the decentralized representation above, without altering the identification results, which rely only on its additive separability from remaining full income.

To summarize, we have the following result.

Proposition 6. *Under Assumptions A.1 and A.3, and the conditions of Proposition 2, the full cost of children is identified from the observation of a single reduced-form full leisure demand function F_I as function of (w_W, w_H, Y) , provided that either: (i) Assumption A.2 holds and the function $F_I(w_W, w_H, \cdot)$ is locally invertible with respect to remaining full income; or (ii) Assumption A.2' holds and the corresponding marginal-response function $D_I(w_W, w_H, \cdot)$ is locally invertible.*

As discussed above, this reduced-form strategy is robust to a broad range of interactions between spouses' choices that need not be explicitly modeled or structurally identified. A further advantage is that identification requires only one spouse's full leisure demand function. Estimating both spouses' demand functions may nevertheless improve precision and yields overidentifying restrictions that can be used for specification testing.

3 Data and Empirical Specification

In this section, we first provide an overview of the data and then specify functional forms for the childcare technology and individual preferences.

3.1 Data

This subsection outlines the sample selection process and describes the variables used in the estimation. We also provide some preliminary, intuitive results based on reduced-form equations.

3.1.1 Sample selection

Our empirical analysis relies on the 2019 wave of the Panel Study of Income Dynamics (PSID), a longitudinal survey that provides detailed information on a broad range of topics, including income and wealth sources, wage rates, working hours, and socio-demographic characteristics. This wave also includes data on household members' time allocation — covering leisure, personal

care, and childcare¹⁹ — as well as disaggregated household expenditures, notably parents’ spending on external childcare services. The 2019 wave covers 9,569 households. For our analysis, we focus on non-student heterosexual couples aged 22 to 50, with and without children and exclude households with more than two children and those in which the youngest child is older than six years. Because our empirical strategy relies on observed wages, we further restrict the sample to dual-earner couples. This latter restriction, in particular, reduces the sample by about 17%, or 20% when considering only couples with children.²⁰ Our final sample comprises 825 households: 463 with children and 362 without.

3.1.2 Key variables

The period of analysis is one week, and all variables are calibrated accordingly. The estimation of the model relies on two key time-related variables: parental childcare time and leisure time. The PSID measures parental childcare time using the following question: “In a typical week, how many hours [do you/does [he/she]] spend caring for or looking after children?” This definition is intentionally broad, encompassing time spent caring for children even when it occurs alongside other activities — including sleep. Consequently, reported childcare time can reach up to 168 hours per week, as documented in Table 1. These figures do not allow for an accurate assessment of the actual time cost of children, since a substantial share of this reported time may coincide with low-effort tasks and impose little to no opportunity cost on parents.²¹ To address this issue,

¹⁹In the PSID, time use information is collected through retrospective questions, rather than time diaries. Nevertheless, Insolera, Johnson, and Simmert (2019) find that time use measures from the PSID are generally consistent with those reported in the American Time Use Survey (ATUS).

²⁰Restricting the sample to dual-earner couples may induce selection if non-working parents differ systematically in childcare burdens or preferences for parental care. However, we focus on ages at which labor-force participation is high, so the resulting selection is relatively limited. Our estimates should nevertheless be interpreted as applying primarily to dual-earner couples.

²¹The original PSID measures reported in Table 1 are consistent with ATUS measures when primary and secondary childcare activities are combined. According to published ATUS tables for 2015–2019 (U.S. Bureau of Labor Statistics, 2019, Tables 9 and 10), total primary and secondary childcare amounts to about 39.2 hours per week for men and 63.3 hours for women living in households with at least one child under age 6. This is also

we adjust the reported values downward to match more closely the American Time Use Survey (ATUS) measures of primary childcare time. If this correction takes the form of subtracting a constant from all observations, it should not greatly affect the estimation of marginal prices, which depend on variation at the margin rather than on absolute levels, but it might generate negative values for some parents. To avoid this eventuality, we apply the following non-linear transformation:

$$t_I^{adj} = t_I^{psid} - k_I \times \tanh\left(\frac{t_I^{psid}}{k_I}\right)$$

where t_I^{psid} is number of hours of child care time as reported by the PSID. Before applying the transformation, we bound this measure so that the sum of weekly childcare and market work does not exceed the 168 hours available in a week. The constant k_I are chosen so that the average adjusted childcare time matches the average values reported in the ATUS.²² This transformation has two desirable properties:²³ it preserves the zero point — i.e., $t_I^{adj} = t_I^{psid}$ when $t_I^{psid} = 0$ — and it approximates a constant downward shift for large values of reported time, since $t_I^{adj} \approx t_I^{psid} - k_I$ when t_I^{psid} is large. As shown in Table 1, the proportion of zero values is negligible and remains unchanged under both definitions, while average childcare time falls substantially under the adjusted measure — to about 10 hours per week for fathers and 19 hours for mothers. Because the adjustment is non-linear, it necessarily changes the distribution of childcare time and may also alter its joint distribution across spouses. We shall therefore assess the sensitivity of our results to this choice by also estimating the model using the original, unadjusted PSID childcare measure; the corresponding results are reported in Section 4.2.1.

consistent with Stewart and Allard (2015), who show that, among families with children, secondary childcare accounts for a large share of total childcare time, with both men and women spending nearly three times more time on secondary than on primary childcare.

²²To do so, we use time use data for men and women with child under 6 from the published ATUS tables for 2015–2019 (U.S. Bureau of Labor Statistics, 2019, Tables 9). In these aggregated data, childcare time refers exclusively to primary activities. It includes physical care, education-related activities, reading to or with children, playing or engaging in hobbies with children, and travel related to caring for and helping household children.

²³Recall that $\tanh(x) = (e^x - e^{-x}) / (e^x + e^{-x})$.

Table 1: Parents’ Weekly Childcare Time (in Hours, with and without Adjustment) and Weekly Childcare Expenditure (in Dollars)

	Mean	S.D.	Min	50%	Max	Pzeros
Father’s Original PSID Childcare Time	35.12	39.72	0.00	20.00	168.00	0.02
Mother’s Original PSID Childcare Time	53.65	46.39	0.00	40.00	168.00	0.01
Father’s Adjusted Childcare Time	9.66	20.65	0.00	0.91	115.17	0.02
Mother’s Adjusted Childcare Time	18.97	28.45	0.00	6.63	116.91	0.01
Childcare Expenditure	99.91	127.46	0.00	49.96	769.23	0.35

Notes: Mean = average value, S.E. = standard deviation, Min = minimum value, 50% = median, Max = maximum value, Pzeros = proportion of zeros.

The second key time-related variable is leisure time. The PSID includes a direct question on usual weekly leisure hours: “In a typical week, how many hours [do you/does [he/she]] spend doing leisure activities for enjoyment, for example, watching TV, doing physical activities that [you enjoy / he or she enjoys], going online, or spending time with friends?” However, this direct measure differs from the notion of leisure used in our model, which is closer to the broader concept of non-market time. The PSID leisure measure excludes many non-market activities — such as personal care, household production, shopping, and sleep — even though these uses of time may also contract when parental time dedicated to childcare increases. This matters because childcare need not crowd out narrowly defined leisure one-for-one; instead, the required time may be drawn from several components of non-market time. A measure based only on reported leisure would therefore miss part of the time reallocation induced by children. For these reasons, we follow the standard practice in the literature and define leisure time as non-market time. Specifically, weekly leisure time is computed as $l_I = T - h_I - t_I$, where T is fixed at 168 hours per week, and h_I denotes hours spent in paid work. Once the $\varphi_I(t_I)$ will be estimated, it will be possible to compute the full leisure time as $L_I = l_I + \varphi_I(t_I) = T - h_I - (1 - \varphi_I(t_I))t_I$.

Finally, the PSID elicits childcare expenditures using the question: “How much did you (and your family living there) pay for child care in 2018?” To obtain weekly childcare expenditures, we divide the reported annual amount by 52. As with other expenditure variables in the PSID, childcare expenditures refer to the previous calendar year, whereas childcare time refers to a

Table 2: Descriptive Statistics of the Benchmark Sample

	Without Child		With Children	
	Mean	S.D.	Mean	S.D.
Father's Adjusted Childcare Time	0.00	0.00	9.66	20.65
Mother's Adjusted Childcare Time	0.00	0.00	18.97	28.45
Childcare Expenditure	0.00	0.00	99.91	127.46
No. of Children	0.00	0.00	1.56	0.50
Child under 3	0.00	0.00	0.74	0.44
Husband's Leisure Time	125.33	12.28	114.76	22.32
Wife's Leisure Time	130.79	12.30	115.49	28.62
Husband's Hourly Wage	30.71	44.51	31.97	25.89
Wife's Hourly Wage	24.36	16.36	29.48	26.25
Full Expenditure (in thousand dollars)	5.52	4.15	6.31	4.25
Husband's Education in Years	14.24	2.26	14.38	2.39
Wife's Education in Years	14.84	2.10	15.15	1.87
Husband's Age in Years	36.71	7.82	34.21	5.33
Wife's Age in Years	35.58	7.84	32.88	5.04

Notes: S.D. = standard deviation. Full expenditure is defined as the sum of household total expenditure on goods and services and expenditure in leisure time computed with spouses' wage rates as prices.

typical week around the time of the interview, creating some temporal mismatch between the two measures. As shown in Table 1, the proportion of zeros is substantial but does not compromise the feasibility of the estimation.

Summary statistics for the variables used in the estimations are presented in Table 2, separately for couples without children and couples with children. On average, childless women report about 15 more hours of leisure per week than mothers, while childless men report around 11 more hours than fathers. For couples with children, average weekly childcare expenditures amount to approximately \$100, with a large standard deviation reflecting substantial heterogeneity in spending. Aside from differences directly related to the presence of children, the two groups of couples exhibit broadly similar average characteristics across the remaining variables. These include spouses' hourly wages (directly provided by the PSID), education levels (measured as the highest grade of school completed), ages (in years). The variables also include the household

full expenditure (defined as the sum of household total expenditure on goods and services and expenditure in leisure time computed with spouses' wage rates as prices) that is used in the estimation instead of the full income to ensure model consistency with a life-cycle framework (Blundell and MaCurdy, 1999).

3.1.3 Preliminary regressions

Before turning to the structural model, we begin by presenting reduced-form OLS estimates of the childcare time and expenditure equations. The estimates are reported in Table 3. The effects of parental hourly wages on childcare expenditures are positive and decreasing, while the effects on childcare time are negative and increasing. These estimates allow for a back-of-the-envelope approximation of the price of parental time devoted to childcare. Evaluated at the average point of the sample (that is, $w_H = 31.97$ and $w_W = 29.48$ using figures in Table 2), a \$10 increase in the father's (mother's) hourly wage is associated with an increase of approximately \$6.44 (\$15.60) in spending on childcare services. At the same time, a \$10 increase in the father's (mother's) hourly wage is associated with a reduction of approximately 0.34 (2.05) hours in the father's (mother's) time devoted to childcare. This implies that the father substitutes one hour of childcare time with \$18.94 ($\approx 6.44/0.34$) of childcare services — interpreted as the value of the father's childcare time. Relative to the value of his wage, this gives $\theta_H = 0.59 \approx 18.94/31.97$. Similarly, the variation in the value of the mother's childcare time is \$7.61 ($\approx 15.60/2.05$), giving $\theta_W = 0.26 \approx 7.61/29.48$.

3.2 Empirical Specification

In this subsection, we present the functional form for the childcare technology and the demands for full leisure. We also incorporate observable and unobservable heterogeneity.

3.2.1 The Childcare Technology

Instead of specifying the childcare technology, we directly model the time cost function. We begin by considering the case of interior solutions.

Table 3: OLS estimations of reduced-form childcare time and expenditure equations

	Expenditure		Father's Time		Mother's Time	
	Est	S.E.	Est	S.E.	Est	S.E.
Constant	31.52	16.72	12.85	2.94	26.31	3.77
Father's Wage	0.74	0.76	-0.04	0.09	-0.05	0.12
Squared Father's Wage / 100	-0.15	0.70	0.01	0.04	0.03	0.05
Mother's Wage	1.92	0.55	-0.10	0.07	-0.27	0.11
Squared Mother's Wage / 100	-0.61	0.33	0.05	0.03	0.11	0.04
Father's Age	3.31	8.41	0.27	1.80	-0.65	2.27
Mother's Age	11.83	9.63	-1.56	1.92	-1.50	2.47
Father's Education	11.08	5.70	-0.77	1.18	-2.63	1.50
Mother's Education	10.18	5.67	-0.15	1.05	0.49	1.73
No. of Children	19.47	5.52	-0.57	1.00	-1.19	1.31
Dummy for Child under 3	6.96	5.11	0.23	1.04	0.33	1.39

Notes: Demographic variables are standardized. Est = estimate, S.E. = standard error.

If $m > 0$ (i.e., the solution is interior), the time cost function is of the Generalized Leontief form (Diewert, 1971) and can be written as follows:

$$e_T = \alpha^* + \beta_W^* w_W^* + \beta_H^* w_H^* + 2\gamma_W \sqrt{w_W^*} + 2\gamma_H \sqrt{w_H^*} + 2\gamma_{WH} \sqrt{w_W^* w_H^*}, \quad (6)$$

where α^* , β_W^* , β_H^* , γ_W , γ_H and γ_{WH} are parameters, w_H^* and w_W^* are the price of parents' productive childcare time as previously explained, which is supposed to be proportionate to hourly wage rates: $w_I^* = \theta_I w_I$ for some positive parameter θ_I . This specification is globally consistent with the theory (increasing and concave with respect to prices) whenever $\beta_W^* \geq 0$, $\beta_H^* \geq 0$, $\gamma_H \geq 0$, $\gamma_W \geq 0$ and $\gamma_{WH} \geq 0$. Its appeal is that it accommodates both substitution and complementarity between parents' inputs through the interaction term $\sqrt{w_W^* w_H^*}$, albeit potentially at the cost of the global consistency.

From Shephard's Lemma, the solution of the cost minimization problem is:

$$t_H = \beta_H^* + \gamma_H \frac{1}{\sqrt{w_H^*}} + \gamma_{WH} \sqrt{\frac{w_W^*}{w_H^*}} \quad (7)$$

$$t_W = \beta_W^* + \gamma_W \frac{1}{\sqrt{w_W^*}} + \gamma_{WH} \sqrt{\frac{w_H^*}{w_W^*}} \quad (8)$$

if $m > 0$. Defining $\beta_I^* = \beta_I \mathbf{z}_{T,I} + v_I$, where $\mathbf{z}_{T,I}$ are control variables, v_I is an error term, and using $w_I^* = \theta_I w_I$, we obtain the childcare time equations:

$$t_H = \beta_H \mathbf{z}_{T,H} + \gamma_H \frac{1}{\sqrt{\theta_H w_H}} + \gamma_{WH} \sqrt{\frac{\theta_W w_W}{\theta_H w_H}} + v_H \quad (9)$$

$$t_W = \beta_W \mathbf{z}_{T,W} + \gamma_W \frac{1}{\sqrt{\theta_W w_W}} + \gamma_{WH} \sqrt{\frac{\theta_H w_H}{\theta_W w_W}} + v_W. \quad (10)$$

To incorporate heterogeneity in the demand for external childcare services, we similarly write: $\alpha^* = \alpha \mathbf{z}_T + u$ where $\mathbf{z}_T$ are control variables and u is an error term. Using $m = e_T - w_H t_H - w_W t_W$ and expressions (6), (9) and (10), we obtain the childcare expenditure equation:

$$m = \begin{cases} m^* & \text{if } m^* \geq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

where

$$m^* = \alpha \mathbf{z}_T + \gamma_W \sqrt{\theta_W w_W} + \gamma_H \sqrt{\theta_H w_H} + u \quad (12)$$

If $m = 0$ (i.e., the solution is at a corner), the parents' childcare time equations switch regime and the prices of parents' productive childcare time must satisfy the following condition:

$$\alpha \mathbf{z}_T + \gamma_W \sqrt{\nu w_W^*} + \gamma_H \sqrt{\nu w_H^*} + u = 0, \quad (13)$$

for some function ν , to guarantee that the childcare constraint is satisfied. This equation can be inverted to recover ν . The shadow prices νw_W^* and νw_H^* are then substituted into (9) and (10).

Finally, the error terms (u, v_W, v_H) are assumed to be jointly normally distributed with mean zero and an unconstrained variance matrix. In our estimations, the control variables $\mathbf{z}_{T,H}$, $\mathbf{z}_{T,W}$ and $\mathbf{z}_T$ include a constant, the spouses' age, the spouses' education, the number of children, as

well as a dummy for the presence of children under the age of 3.

3.2.2 The Full Leisure Demand Equations

To begin with, we note that $\varphi_I(t_I) = (1 - \theta_I)t_I$ in spouses' utility functions since θ_I is constant, as previously mentioned. As a result, the total leisure enjoyed by each spouse is defined as $L_I = l_I + (1 - \theta_I)t_I$. With a linear specification, the prices of productive and leisure-like (or labor-like) childcare time have a simple interpretation: the term $\theta_I t_I$ can be seen as the fraction or percentage of childcare time that is perceived as pure labor, while the term $(1 - \theta_I)t_I$ as the fraction that is perceived as pure leisure. The linear specification also guarantees that the utility profit functions π_W and π_H are always equal to zero.

As previously discussed, our empirical model employs full expenditure — denoted as Y — rather than full income as an explanatory variable. The time cost of children is derived from the childcare technology and defined by expression (6), while the monetary cost is specified as:

$$e_M = \log(1 + \exp(\kappa_0 + \kappa_H w_H + \kappa_W w_W + \kappa_Y Y)) \times \sqrt{n} \quad (14)$$

where κ_0 , κ_H , κ_W , and κ_Y are parameters, and n is the number of children. The transformation in (14) is commonly referred to as the softplus function in machine learning. It ensures that monetary costs remain strictly positive, while offering a smooth and differentiable alternative to the simple maximum operator or the exponential function.

The remaining full expenditure Y_R is equal to Y for couples without children and to $Y - e_M - e_T$ for couples with children. The specification of e_M is therefore substituted directly into Y_R and hence into the full leisure demand equations. For couples with and without children, the full leisure demand equations are specified either as:

(i) a quadratic function of remaining full expenditure:

$$L_H = \mathbf{a}_H \mathbf{z}_{L,H} + b_H w_H + c_H w_W + d_H Y_R + e_H Y_R^2 + \varepsilon_H \quad (15)$$

$$L_W = \mathbf{a}_W \mathbf{z}_{L,W} + b_W w_W + c_W w_H + d_W Y_R + e_W Y_R^2 + \varepsilon_W \quad (16)$$

where $\mathbf{a}_I$, b_I , c_I , d_I and e_I are parameters while $\mathbf{z}_{L,I}$ are control variables, and ε_I is an error term, or as:

(ii) a quadratic function of remaining full expenditure with a demographic shift:

$$L_H = \mathbf{a}_H(n)\mathbf{z}_{L,H} + b_H(n)w_H + c_H(n)w_W + d_H Y_R + e_H Y_R^2 + \varepsilon_H \quad (17)$$

$$L_W = \mathbf{a}_W(n)\mathbf{z}_{L,W} + b_W(n)w_W + c_W(n)w_H + d_W Y_R + e_W Y_R^2 + \varepsilon_W \quad (18)$$

where $\mathbf{a}_I(n) = \mathbf{a}_I + \mathbf{a}'_I \delta(n)$, $b_I(n) = b_I + b'_I \delta(n)$ and $c_I(n) = c_I + c'_I \delta(n)$ are functions of n , $\mathbf{a}'_I$, b'_I and c'_I are parameters and $\delta(n)$ is a dummy variable equal to one if $n > 0$ and 0 otherwise. This specification allows the presence of children to shift the full leisure demand function, while leaving the slope of the Engel curves unchanged.

In our estimations, the control variables $\mathbf{z}_{L,H}$ and $\mathbf{z}_{L,W}$ include a constant, spouses' education and spouses' age. The error terms $(\varepsilon_W, \varepsilon_H)$ are assumed to have mean zero and an unconstrained variance matrix.

4 Estimation Method and Empirical Results

4.1 Estimation Method

In this subsection, we first present the two stage estimation method based on the Maximum Likelihood method and then the Three-Stage Least Squares method.

While the five equations could be estimated simultaneously, we simplify the estimation procedure by taking advantage of the two-stage decision-making structure and estimate the system recursively. A benefit is that potential specification errors are not conflated across stages, implying that price-scale coefficients can be estimated separately. Moreover, the two stages rely on different samples: the first-stage is estimated using only couples with children, while the second-stage is estimated using the full sample of couples. In the first-stage, the three-equation system (9)–(12) for the childcare technology is estimated. In the data, a substantial proportion

of parents report zero expenditure on paid childcare services, whereas the number of parents reporting zero hours of childcare time per week is negligible and may reflect measurement errors. We therefore assume that the two equations for parental childcare time are uncensored, whereas the equation for childcare expenditure is censored at zero. For tractability, we do not model the regime switch induced by zero childcare expenditure in the childcare-time equations, as doing so would introduce substantial additional nonlinearity. The resulting system is thus a mixed specification, combining linear and Tobit components, and is estimated by the Maximum Likelihood (ML) method. In the second-stage, the time cost and the value of both full leisure times are computed for all observations using $\hat{\theta}_W$ and $\hat{\theta}_H$ obtained in the first-stage. The two-equation system (15)–(16) or (17)–(18) for spouses’ full leisure demand is estimated by the Three-Stage Least Squares (3SLS) method. This estimation method accounts for the potential endogeneity of full expenditure and the time cost computed in the first-stage problem. Such endogeneity may indeed arise for several reasons. Most notably, measurement errors in working time and childcare time — and, by construction, in leisure time — are directly reflected in the computed values of full expenditure and childcare cost, respectively. To address this issue, we replace both potentially endogenous variables with predicted counterparts. Although full expenditure and the time cost enter the second-stage equations only through their sum, we construct their predicted values separately. The predicted time cost is directly obtained from the first-stage estimates by setting the corresponding error term equal to zero. Full expenditure, by contrast, is predicted using an auxiliary OLS regression on a constant, spouses’ education, spouses’ age, spouses’ hourly wages, weekly nonlabor income, and its square, with weekly nonlabor income and its square serving as the excluded instruments. This auxiliary regression is run separately for couples with and without children. While nonlabor income is a standard source of exogenous variation in labor supply and demand models, as emphasized for instance by Blundell and MaCurdy (1999), the predicted time cost provides an additional and source of variation specific to the model. It captures variation in the opportunity cost of childcare time implied by the first-stage time-allocation equations and therefore contributes directly to the predicted full cost measure used in the second-stage.

4.2 Empirical Results

4.2.1 Estimates of the Childcare Equations

In Table 4, the estimated parameters for the childcare time and expenditure equations are reported for five different samples or specifications. The estimates and standard errors in columns A are based on the sample described in Table 2 and those in columns B are based on a restricted sample constructed by dropping the 10 largest and 10 smallest observations for spouses' hourly wages. This restriction aims at mitigating the potential influence of outliers and ensures that estimates reflect general patterns rather than a few extreme cases. The estimates and standard errors in columns C are based on an extended sample which includes all couples without restriction on the number of children. The estimates and standard errors in columns D are based on a sample that excludes couples with zero childcare expenditure to assess the robustness when identification relies only on observations with positive childcare expenditure. The estimates and standard errors in columns E are based on the same sample as for columns A except that it uses the original PSID childcare time measures rather than the adjusted ones. Overall, the results are similar across samples, although the parameter estimates are generally imprecise. The intercepts in the childcare-time equations are much larger in columns E because they absorb the constants k_H and k_W used in the construction of adjusted childcare times. In columns C, where all couples are included regardless of the number of children, the estimated effects of the child-related variables naturally change. These differences notwithstanding, most control variables are not statistically significant, with a few exceptions.

As expected, a higher price of productive childcare time, w_H^* or w_W^* , is associated with less time spent on childcare and greater expenditure on paid childcare services. More precisely, the derivative of the father's childcare time equation with respect to the price of his productive childcare time is given by:

$$\frac{\partial t_H}{\partial w_H^*} = -\frac{1}{2}\gamma_H (w_H^*)^{-3/2} - \frac{1}{2}\gamma_{HW} (w_W^*)^{1/2} (w_H^*)^{-3/2}.$$

Using the estimates in column A and evaluating the derivative at the median of the wage

Table 4: ML Estimates of Childcare Time and Expenditure Equations

		A		B		C		D		E	
Variable		Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
Childcare Expenditure Equation											
α_0	Intercept	-99.28	28.71	-100.91	34.90	-136.65	25.19	50.10	25.63	-98.37	28.71
α_1	Husband's Educ.	15.82	8.99	16.82	8.31	11.00	8.04	11.13	7.66	15.59	9.01
α_2	Wife's Educ.	15.16	9.41	5.32	8.73	12.04	8.53	13.39	7.95	15.34	9.41
α_3	Husband's Age	6.99	13.19	16.04	11.90	-9.63	10.77	-1.67	11.22	7.00	13.19
α_4	Wife's Age	19.56	13.77	8.18	12.51	31.09	11.02	13.07	11.60	19.31	13.76
α_5	Child under 3	9.40	8.08	2.57	7.35	7.29	6.67	15.06	7.04	9.42	8.07
α_6	No. of Children	30.08	7.87	22.27	7.14	-12.27	7.09	11.42	6.81	30.05	7.86
Father's Childcare Time Equation											
$\beta_{H,0}$	Intercept	6.07	2.35	5.58	3.09	3.20	2.15	3.70	3.43	27.40	4.62
$\beta_{H,1}$	Husband's Educ.	-0.53	1.13	0.04	1.19	0.24	0.98	0.75	1.22	-0.70	2.18
$\beta_{H,2}$	Wife's Educ.	0.01	1.18	0.22	1.25	0.81	1.03	0.46	1.31	0.11	2.26
$\beta_{H,3}$	Husband's Age	0.25	1.64	0.46	1.70	-2.14	1.30	0.78	1.78	0.07	3.15
$\beta_{H,4}$	Wife's Age	-1.32	1.71	-1.65	1.78	0.44	1.34	-0.67	1.86	-2.79	3.29
$\beta_{H,5}$	Child under 3	0.31	1.01	0.97	1.05	0.63	0.82	1.28	1.12	1.63	1.93
$\beta_{H,6}$	No. of Children	-0.56	0.97	0.09	1.01	0.08	0.84	-0.02	1.08	-0.13	1.87
Mother's Childcare Time Equation											
$\beta_{W,0}$	Intercept	12.95	2.52	8.14	4.79	14.39	2.36	6.43	4.07	44.72	4.10
$\beta_{W,1}$	Husband's Educ.	-2.53	1.54	-1.62	1.58	-1.08	1.45	-1.76	1.51	-3.96	2.51
$\beta_{W,2}$	Wife's Educ.	-0.05	1.56	0.19	1.67	0.40	1.51	1.06	1.59	0.72	2.56
$\beta_{W,3}$	Husband's Age	-0.33	2.24	1.29	2.28	-2.51	1.94	0.05	2.23	-0.88	3.67
$\beta_{W,4}$	Wife's Age	-1.73	2.32	-2.89	2.37	0.24	1.98	-0.84	2.31	-2.92	3.80
$\beta_{W,5}$	Child under 3	0.40	1.38	1.08	1.40	1.39	1.22	0.67	1.41	0.67	2.26
$\beta_{W,6}$	No. of Children	-1.17	1.33	-0.37	1.35	2.11	1.25	0.50	1.37	-1.90	2.17
Wage Coefficients											
γ_H		20.51	6.67	22.44	7.41	28.39	5.90	10.53	6.86	27.50	9.85
γ_W		27.32	6.38	36.77	9.44	28.72	5.96	22.43	8.38	31.45	8.59
$\gamma_{W,H}$		-3.26	2.46	-4.84	2.90	-3.63	2.39	0.18	3.00	-4.07	4.61
Price-Scale Coefficients											
θ_H		0.35	0.21	0.28	0.20	0.34	0.13	0.20	0.35	0.20	0.12
θ_W		0.44	0.26	0.30	0.14	0.42	0.24	0.39	0.25	0.32	0.24

Notes: Demographic variables are standardized. A = benchmark sample (463 observations); B = sample excluding extreme values (413); C = sample including couples with more than two children (681); D = sample excluding couples with zero childcare expenditure (303); E = sample using the original PSID childcare time measures (463).

distributions, this corresponds to a marginal effect of approximately -0.18 (S.E. = 0.11). Thus, a one-dollar increase in the price of productive childcare time is associated with about 0.18 fewer hours of childcare per week or approximately 11 minutes. The effect is moderate at the median but strongly nonlinear. At the first decile of the men’s wage distribution, the marginal effect is about -0.65 hours (S.E. = 0.38) whereas at the ninth decile it falls to about -0.05 (S.E.=0.03). Similarly, the derivative with respect to the mother’s price is given by:

$$\frac{\partial t_H}{\partial w_W^*} = \frac{1}{2} \gamma_{HW} (w_H^*)^{-1/2} (w_W^*)^{-1/2}$$

Evaluated at the median of the wage distributions, the marginal effect is equal to -0.17 (S.E. = 0.13) or approximately 10 minutes. The negative cross-wage effect implies that the two parental time inputs can be seen as Hicks–Allen complements. Finally, the derivative of the mother’s childcare time equation with respect to her price to -0.27 hours (S.E. = 0.09) or approximately 16 minutes while the derivative with respect to the father’s price is, by symmetry, also equal to -0.17 (S.E. = 0.13).²⁴ The fact that the parameter γ_{WH} is negative implies that the cost function is not globally consistent.

The most important parameters in the model are the price-scale coefficients θ_H and θ_W . Under the linear specification of the process benefits, these parameters can be interpreted as the proportion of childcare time perceived as pure labor by the mother and the father, respectively. Using the benchmark sample (columns A), our results suggest that about 44% of the time mothers spend caring for their children is treated as labor, while the remainder resembles leisure. This proportion is lower for fathers and equal to about 35%. Using the sample which trims extreme

²⁴The number of studies that explicitly examine the relationship between childcare time and hourly wages is very limited. They generally do not find significant effects or confirm the negative effect we obtain (Bloemen and Stanca, 2014, for France; Kalenkoski, Ribar, Stratton, 2009, for U.K.; Hallberg and Klevmarken, 2003, and Gustafsson and Kjulin, 1994, for Sweden). Among the very few available studies based on U.S. data, Kimmel and Connelly (2007) present the surprising finding, with the American Time Use Survey (ATUS), that an increase in wage rates is associated with an increase in time spent on childcare. They interpret this as an income-effect mechanism: as wages rise, households demand higher-quality childcare activities. Note that their sample includes both couples and single-parent households, with multiple and older children.

values (columns B), the couples with positive childcare expenditure (columns D) or the original PSID childcare time measures (columns E), the coefficients are somewhat lower but not significantly different. Overall, the estimated price-scale coefficients are broadly similar in magnitude to the back-of-the-envelope approximations obtained from preliminary OLS estimations.

To assess robustness, we also re-estimate models with a more parsimonious set of covariates; the results change little even if the price-scale coefficients are slightly larger. We also fit a more flexible cost function that includes quadratic terms in hourly wage; the added terms are not statistically different from zero at the 5% level. We finally test the exclusion of full expenditure from the childcare equations using a likelihood-ratio (LR) test. Since full expenditure is potentially endogenous, we replace it with its natural instrument, non-labor income, which is also used as an instrument in the estimation of the full leisure demand equations below. The joint null hypothesis that the three coefficients on non-labor income in the childcare equations are zero is not rejected at conventional levels. Overall, these diagnostics provide no strong evidence against the exclusion restriction.

That said, the estimated price-scale coefficients might be subject to downward bias for at least two reasons. First, the demand for formal childcare services may be rationed — either because the couple lives in a region with limited availability of childcare facilities, or because available services do not match the parents’ preferences in terms of quality or schedule. Second, the actual cost of using formal childcare services may be higher than what is reported in the PSID, as it may involve fixed, unobserved costs. These factors could attenuate the sensitivity of the demand for childcare services and lead to an underestimation of price-scale coefficients. Despite these potential limitations, however, the estimates appear plausible and provide a solid basis for computing both spouses’ full leisure time and the time cost of children.

4.2.2 Estimates of the Full Leisure Demand Equations

In Table 5, the estimates and standard errors for the full leisure demand equations are reported for four different samples or specifications. The estimates and standard errors in columns A are obtained from a sample of childless couples and thus do not involve the estimation of

the monetary cost of children. The estimates and standard errors in columns B and C correspond, respectively, to the standard Quadratic Engel curve specification and the Quadratic, Demographic-Shift Engel curve specification. The estimates and standard errors in columns D correspond to the same specification as in columns B with an alternative definition of full leisure. In this definition, the time devoted to domestic chores is subtracted from the previous measure. The bottom panel of the table reports the estimated parameters of the monetary cost function. Using the sample of childless couples, we find a positive but weakly significant relationship between full expenditure and full leisure. A Wald test does not reject the null hypothesis of a flat Engel curve in both equations ($p\text{-value} = 0.11$). When couples with children are included in the sample, the estimates remain essentially unchanged but the standard errors are reduced. It reflects the larger sample, greater variation in full leisure and full expenditure, and the strong instrument provided by the fitted time cost. Consequently, the parameters associated with Engel curves in columns B are generally significantly different from zero, with the notable exception of the quadratic terms. In particular, a \$1,000 increase in full expenditure is associated (ignoring quadratic terms to simplify) with an additional 3.2 hours of full leisure for men and 4.4 hours for women, while higher hourly wages are associated with less full leisure for both spouses. Column C of Table 5 report estimates and standard errors for the Quadratic, Demographic-Shift Engel curve specification. The results remain very similar overall, although standard errors are somewhat larger, and none of the additional demographic-shift terms is statistically significant at conventional levels. Nevertheless, the signs of the estimated shifts are informative: the fact that both intercept shifts are negative suggests that part of the effect of children is absorbed by these shifts rather than by the monetary cost function. The estimates based on an alternative measure of leisure, reported in columns D, are very similar to those in columns B, although the estimated effects of full expenditure are slightly larger.

Three final remarks are in order. First, the parameters of the monetary cost function are imprecisely estimated and adding covariates substantially increases standard errors. Second, because the model is overidentified and the monetary cost parameters can be identified from either spouse's full leisure equation alone, we also estimate a specification allowing these parameters to differ across spouses. Using the standard Quadratic Engel curve specification, a distance test

Table 5: 3SLS Estimates of Full Leisure Demand Equations

		A		B		C		D	
Param.	Variables	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
Husband' Engel Curve									
$a_{H,0}$	Intercept	123.25	3.86	120.19	2.71	120.30	2.80	119.95	3.84
$a_{H,1}$	Husband's Educ.	-1.03	0.83	-1.50	0.56	-1.14	0.86	-1.53	0.80
$a_{H,2}$	Wife's Educ.	0.66	0.75	0.35	0.57	0.53	0.78	0.04	0.82
$a_{H,3}$	Husband's Age	-0.53	1.14	-0.75	0.89	-0.64	1.20	-0.56	1.26
$a_{H,4}$	Wife's Age	-0.07	1.11	0.23	0.91	-0.04	1.17	0.43	1.30
b_H	Husband's Wage	0.04	0.25	-0.29	0.14	-0.13	0.18	-0.53	0.20
c_H	Wife's Wage	-0.10	0.23	-0.30	0.13	-0.25	0.16	-0.50	0.19
d_H	Full Expend.	0.58	2.48	3.24	1.43	2.52	1.60	4.81	1.98
e_H	Sq. Full Expend.	-0.00	0.04	0.00	0.03	-0.02	0.03	0.03	0.05
Husband's Demographic Shift									
$a'_{H,0}$	Intercept	.	.	.	.	-1.80	3.07	.	.
$a'_{H,1}$	Husband's Educ.	.	.	.	.	-0.59	1.14	.	.
$a'_{H,2}$	Wife's Educ.	.	.	.	.	-0.31	1.16	.	.
$a'_{H,3}$	Husband's Age	.	.	.	.	-0.14	1.79	.	.
$a'_{H,4}$	Wife's Age	.	.	.	.	0.81	1.86	.	.
b'_H	Husband's Wage	.	.	.	.	-0.10	0.06	.	.
c'_H	Wife's Wage	.	.	.	.	0.11	0.06	.	.
Wife's Engel Curve									
$a_{W,0}$	Intercept	129.03	3.83	127.65	3.40	128.59	3.53	127.12	5.26
$a_{W,1}$	Husband's Educ.	-0.64	0.82	-0.05	0.62	-0.71	0.95	0.58	0.99
$a_{W,2}$	Wife's Educ.	-0.72	0.74	-1.47	0.64	-0.69	0.86	-2.36	1.03
$a_{W,3}$	Husband's Age	-0.56	1.13	-1.50	0.97	-0.50	1.31	-1.47	1.55
$a_{W,4}$	Wife's Age	0.27	1.10	0.76	1.01	0.28	1.28	1.30	1.62
b_W	Husband's Wage	-0.46	0.25	-0.46	0.19	-0.47	0.21	-0.88	0.29
c_W	Wife's Wage	-0.56	0.23	-0.50	0.17	-0.35	0.23	-0.85	0.27
d_W	Full Expend.	4.29	2.46	4.40	1.89	3.72	2.19	7.10	2.89
e_W	Sq. Full Expend.	0.03	0.04	0.02	0.04	0.01	0.04	0.07	0.06
Wife's Demographic Shift									
$a'_{W,0}$	Intercept	.	.	.	.	-3.19	3.94	.	.
$a'_{W,1}$	Husband's Educ.	.	.	.	.	1.24	1.27	.	.
$a'_{W,2}$	Wife's Educ.	.	.	.	.	-1.29	1.33	.	.
$a'_{W,3}$	Husband's Age	.	.	.	.	-2.07	1.98	.	.
$a'_{W,4}$	Wife's Age	.	.	.	.	0.94	2.07	.	.
b'_W	Husband's Wage	.	.	.	.	0.08	0.08	.	.
c'_W	Wife's Wage	.	.	.	.	0.02	0.07	.	.
Monetary Cost									
κ_0	Intercept	.	.	3.31	2.27	3.81	3.47	4.89	2.57
κ_H	Husband's Wage	.	.	-0.17	0.13	-0.18	0.21	-0.14	0.12
κ_W	Wife's Wage	.	.	-0.08	0.14	-0.02	0.21	-0.06	0.12
κ_Y	Full Expend.	.	.	0.45	1.31	0.14	2.05	0.11	1.18

Notes: Demographic variables are standardized. Observations with full expenditure exceeding \$20,000 per week are excluded. A = childless sample (353 observations); B = Quadratic Engel curve specification (816); C = Quadratic, Demographic-Shift Engel curve specification (816); D = sample with alternative definition for leisure (816).

does not reject the hypothesis that the parameters are the same for fathers and mothers (p-value = 0.98). Third, we also assess the exclusion restriction on nonlabor income by re-estimating the standard Quadratic Engel curve specification with nonlabor income and its square entering directly into both spouses' full leisure equations, and make a Wald test of the joint null that the four additional coefficients are zero. The null hypothesis is not rejected (p-value = 0.98).

4.2.3 Estimates of the time and monetary cost of children

The characteristics of the distribution of time and monetary costs, across the different specifications and for one and two children compositions, are reported in Table 6.²⁵ The median time cost for a single child is sizable — about \$230 per week (row A), \$170 when trimming extreme values (row B), \$220 when using the extended sample with more than two children (row C), \$230 when excluding couples with zero childcare expenditure (row D), and \$570 when using the original PSID childcare time measure (row E) — representing between 7% and 25% of total spousal labor income. The differences between the first four specifications are mainly driven by variations in the price of productive childcare time. The fifth specification relies on a time measure that includes passive childcare which likely leads to an overstatement of the true time cost (even if the estimated price of productive childcare time is somewhat lower). By comparison, when parental childcare time is instead valued at observed wages (row A*), the median time cost of a single child rises to about \$350 per week, roughly 50% above the benchmark estimate based on adjusted shadow prices. The density function of time costs under the benchmark specification (row A), pooling the sample of couples with one and two children, is shown in Figure 1. The distribution is markedly right-skewed, with a long upper tail, so mean values exceed medians substantially and are likely less representative of a typical household.

Finally, the monetary cost for a single child, evaluated at its sample median, is estimated at

²⁵The following discussion focuses on the distribution of time and monetary costs and does not report standard errors. Standard errors for costs evaluated at any given point of the distribution can be obtained using the delta method. For example, we evaluate the cost at the median of value of hourly wages and compute the corresponding standard error. The resulting time and monetary costs are approximately \$344 (S.E. = 120) and \$566 (S.E. = 286), respectively. These estimates are relatively imprecise and only marginally statistically different from zero.

Table 6: Descriptive Statistics for the Distribution of the Weekly Time and Monetary Cost of Children (in Thousands of Dollars per Week)

Children		count	mean	std	min	25%	50%	75%	max
Weekly time cost in thousands dollars									
A	1	205.00	0.43	0.64	0.00	0.08	0.23	0.52	5.14
	2	258.00	0.40	0.50	0.00	0.10	0.25	0.48	3.63
B	1	183.00	0.29	0.36	0.00	0.07	0.17	0.36	2.58
	2	230.00	0.29	0.36	0.00	0.08	0.20	0.36	2.87
C	1	205.00	0.42	0.61	0.00	0.08	0.22	0.51	4.94
	2	258.00	0.38	0.48	0.00	0.09	0.24	0.47	3.50
D	1	117.00	0.36	0.50	0.02	0.11	0.23	0.39	4.41
	2	186.00	0.38	0.40	0.02	0.14	0.27	0.45	2.58
E	1	205.00	0.81	0.90	0.00	0.33	0.57	0.97	8.07
	2	258.00	0.76	0.73	0.00	0.32	0.56	0.94	6.15
A*	1	205.00	0.93	1.54	0.00	0.14	0.35	1.08	11.47
	2	258.00	0.80	1.21	0.00	0.15	0.39	0.90	9.83
Weekly monetary cost in thousands dollars									
B	1	203.00	0.77	0.80	0.00	0.09	0.48	1.25	2.95
	2	252.00	1.12	1.11	0.00	0.20	0.79	1.76	4.53
C	1	203.00	0.80	0.88	0.00	0.06	0.43	1.28	3.67
	2	252.00	1.13	1.20	0.00	0.12	0.71	1.73	4.88
D	1	203.00	1.21	1.12	0.00	0.18	0.89	2.04	3.96
	2	252.00	1.79	1.56	0.00	0.39	1.40	2.86	6.13

Notes: Computations do not use PSID sample weights. For time cost, A = benchmark sample, B = sample excluding extreme values, C = sample including couples with more than two children, D = sample excluding couples with zero childcare expenditure, E = sample using the original PSID childcare time measures, A* = benchmark sample with childcare time valued at observed wages. For monetary cost, B = Quadratic Engel curve specification, C = Quadratic, Demographic-Shift Engel curve specification, D = alternative definition of leisure.

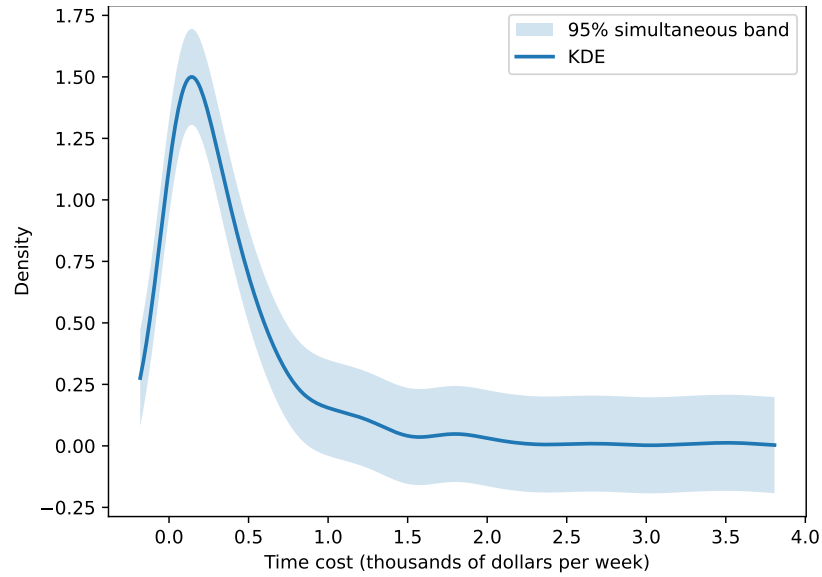


Figure 1: Distribution of Time Costs (in thousands of dollars per week)

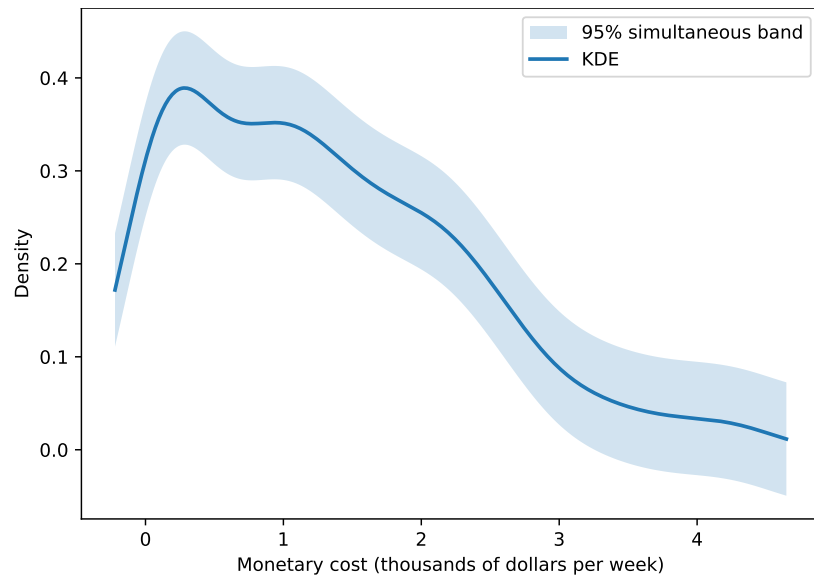


Figure 2: Distribution of Monetary Costs (in thousands of dollars per week)

Notes: Computations do not use PSID sample weights. The solid lines report kernel density estimates of the distributions of estimated costs. The shaded areas are 95% simultaneous bootstrap confidence bands for the densities, based on 1,000 resamples of households with model parameters held fixed. They therefore do not incorporate the uncertainty of parameter estimation.

around \$480 per week for the standard Quadratic Engel curve specification (row B), \$430 per week for the Quadratic, Demographic-Shift Engel curve specification (row C), and \$890 per week for the sample with the alternative definition of leisure (row D). The density function of monetary costs under the Quadratic Engel curve specification (row B) is shown in Figure 2.

5 Conclusion

In this paper, we have developed a static model to evaluate the full cost of children, including both a monetary component and a time component. A crucial point of our study lies in recognizing that the price of time used to assess the time cost of children is not necessarily equivalent to the parents' wage rate. Instead, it depends on how parents perceive their time spent with their children — whether they consider it closer to leisure or to labor. To the best of our knowledge, this is the first study to explicitly make this distinction and to provide estimates of the full cost of children that account for this nuanced valuation of time.

Estimating the model requires detailed information on both the time parents devote to their children and their spending on external childcare. We implement it using the 2019 PSID, which provides the necessary data. Recovering the full cost of children raises two challenges, though. First, the full leisure demand equations are only weakly responsive to changes in full expenditure so identification based on leisure alone is potentially weak. Importantly, our procedure first recovers the time cost and then the full cost of children, with the monetary cost obtained as the difference between the two. This sequence is not essential, however. Alternative implementations could rely on adult goods with steeper Engel curves, such as clothing, either to recover the monetary component — which would then be added to the independently identified time component to obtain the full cost — or, under suitable additional conditions, to identify the full cost directly. Exploring these alternatives would therefore require a substantially different empirical strategy and is left for future research. Second, full expenditure must be instrumented, and credible instruments are scarce. Our theoretical framework nonetheless delivers a strong instrument — the fitted value of the child time cost obtained from the childcare equations — although it is model-dependent.

Notwithstanding these caveats, our baseline results indicate that one hour of the mother’s productive childcare time is priced at approximately 44% of her hourly wage, and one hour of the father’s at 35%. The prices of productive childcare time differ from current hourly wages because a fraction of childcare time can be assimilated to pure leisure. The resulting time cost of children is substantial: for a single child, its median is about \$230 per week. The corresponding median monetary cost is approximately \$620 per week, implying a total cost in excess of \$800 per week, well above typical estimates in the literature. These figures, moreover, capture only short-run costs. They exclude longer-term consequences such as career interruptions and reduced human capital accumulation, particularly for mothers, so the overall economic cost of children may be considerably larger.

Beyond these estimates, the methodology developed in this paper has several potential applications. By operationalizing Reid’s (1934) third-party criterion, it provides a measure of the value of childcare services produced within the household that could otherwise be supplied by the market, and may therefore contribute to household satellite accounts and broader measures of national production. In particular, it offers an alternative to the common practice of valuing all parental childcare time at the market wage, which may substantially overstate the value of household childcare production when parental time also generates direct utility. For practical applications in which the full model cannot be estimated, our results suggest that valuing parental childcare time at about 30–50 percent of the wage may provide a useful rule of thumb and a more realistic benchmark than using the full wage rate.

The resulting measure of the full cost of children can also be used in welfare analysis, for instance to construct equivalence scales or assess poverty and inequality after accounting for both monetary expenditures and parental time costs. More specifically, the structural framework can support the evaluation of policies that alter the relative cost of parental and purchased childcare, such as childcare subsidies, tax credits, or public daycare provision. Such policies may affect not only household expenditures on childcare but also the allocation of care between parents and the market. By distinguishing the monetary and time components of the cost of children, the framework makes it possible to evaluate these adjustments without mechanically valuing every change in parental childcare time at the wage rate.

Appendix A : Proofs

Proof of Proposition 1.

The first order conditions of the optimization problem $\bar{P}$ are:

$$l_W : \phi_W \frac{\partial U_W}{\partial l_W} - \lambda_1 w_W = 0 \quad (\text{A.1})$$

$$t_W : \phi_W \frac{\partial U_W}{\partial t_W} - \lambda_1 w_W - \lambda_2 \frac{\partial g}{\partial t_W} = 0 \quad (\text{A.2})$$

$$c_W : \phi_W \frac{\partial U_W}{\partial c_W} - \lambda_1 = 0 \quad (\text{A.3})$$

$$l_H : \phi_H \frac{\partial U_H}{\partial l_H} - \lambda_1 w_H = 0 \quad (\text{A.4})$$

$$t_H : \phi_H \frac{\partial U_H}{\partial t_H} - \lambda_1 w_H - \lambda_2 \frac{\partial g}{\partial t_H} = 0 \quad (\text{A.5})$$

$$c_H : \phi_H \frac{\partial U_H}{\partial c_H} - \lambda_1 = 0 \quad (\text{A.6})$$

$$c_K : \phi_H \frac{\partial U_H}{\partial c_K} + \phi_W \frac{\partial U_W}{\partial c_K} - \lambda_1 = 0 \quad (\text{A.7})$$

$$m : \lambda_2 - \lambda_1 + \mu = 0 \quad (\text{A.8})$$

where λ_1 and λ_2 are the Lagrange multipliers for the budget constraint (3) and the childcare technology constraint (1), respectively, and μ is the Kuhn-Tucker multiplier for the nonnegativity constraint (2), with the following (dual and complementary slackness) conditions:

$$\mu m = 0 \text{ and } \mu \geq 0. \quad (\text{A.9})$$

Part 1. Define $\lambda = \lambda_2$ and, from (A.8), write $\lambda + \mu = \lambda_1$. Then, from (A.2), we obtain:

$$w_W \cdot \left(1 - \frac{\phi_W}{w_W (\lambda + \mu)} \frac{\partial U_W}{\partial t_W} \right) + \frac{\lambda}{\mu + \lambda} \cdot \frac{\partial g}{\partial t_W} = 0$$

Define $\nu = 1 + \mu/\lambda$ and

$$\theta_W = 1 - \frac{\phi_W}{\lambda \nu w_W} \frac{\partial U_W}{\partial t_W},$$

so that we obtain:

$$w_W \cdot \nu \cdot \theta_W + \frac{\partial g}{\partial t_W} = 0. \quad (\text{A.10})$$

Using (A.1)

$$\theta_W = 1 - \frac{\phi_W}{\lambda \nu w_W} \frac{\partial U_W}{\partial t_W} = 1 - \frac{\partial U_W / \partial t_W}{\partial U_W / \partial l_W} > 0,$$

from the assumptions on utility functions. Similarly, from (A.5), we obtain:

$$w_H \cdot \nu \cdot \theta_H + \frac{\partial g}{\partial t_H} = 0. \quad (\text{A.11})$$

This system of two equations (A.10)–(A.11) constitutes the first-order conditions of the optimization problem P_0 :

$$\begin{aligned} t_W : \theta_W w_W + \frac{\partial g}{\partial t_W} (1 - \zeta) &= 0 \\ t_H : \theta_H w_H + \frac{\partial g}{\partial t_H} (1 - \zeta) &= 0 \end{aligned}$$

where ζ is the Kuhn-Tucker multiplier associated with the constraint $g(t_W, t_H, n) \geq 0$, with $1 - \zeta = 1/\nu = 1/(\lambda + \mu)$.

Part 2. We write: $\lambda_I = \lambda_1/\phi_I$. From (A.1)–(A.3) or (A.4)–(A.6), we obtain:

$$\frac{\partial U_I}{\partial l_I} - \lambda_I w_I = 0, \quad (\text{A.12})$$

$$\frac{\partial U_I}{\partial t_I} - \lambda_I (1 - \bar{\theta}_I) w_I = 0, \quad (\text{A.13})$$

$$\frac{\partial U_I}{\partial c_I} - \lambda_I = 0, \quad (\text{A.14})$$

with $I = W$ and H . Conditional on c_K , this system of three equations (A.12)–(A.14) constitutes the first-order conditions of the optimization problem P_I , where λ_I is the Lagrange Multiplier of the budget constraint.

Part 3. By the envelope theorem, the first-order conditions of Problems P_W and P_H imply:

$$\frac{\partial V_I}{\partial \kappa_I} = \lambda_I, \quad \frac{\partial V_I}{\partial c_K} = \frac{\partial U_I}{\partial c_K},$$

with $I = H, W$. Since $\lambda_I = \lambda_1/\phi_I$, it follows that:

$$\phi_W \frac{\partial V_W}{\partial \kappa_W} = \lambda_1, \quad \phi_H \frac{\partial V_H}{\partial \kappa_H} = \lambda_1,$$

and, from (A.7), it follows that:

$$\phi_H \frac{\partial V_H}{\partial c_K} + \phi_W \frac{\partial V_W}{\partial c_K} - \lambda_1 = 0$$

Hence, the first-order conditions of the optimization problem P_2 are satisfied.

Remark. The proof above assumes that the time-feasibility constraints are slack, so that both spouses supply a strictly positive amount of market work. Suppose instead that spouse I does not work in the market. Then $l_I + t_I = T$ and the corresponding time-feasibility constraint is binding. Let $\eta_I \geq 0$ denote its Kuhn–Tucker multiplier. The first-order conditions with respect

to l_I and t_I become:

$$l_I : \phi_I \frac{\partial U_I}{\partial l_I} - \lambda_1 w_I - \eta_I = 0 \quad (\text{A.15})$$

$$t_I : \phi_I \frac{\partial U_I}{\partial t_I} - \lambda_1 w_I - \lambda_2 \frac{\partial g}{\partial t_I} - \eta_I = 0 \quad (\text{A.16})$$

Define the shadow price of spouse I 's time by $w_I^s = w_I + \eta_I/\lambda_1$. The preceding conditions can then be rewritten as

$$\phi_I \frac{\partial U_I}{\partial l_I} - \lambda_1 w_I^s = 0 \quad (\text{A.17})$$

$$\phi_I \frac{\partial U_I}{\partial t_I} - \lambda_1 w_I^s - \lambda_2 \frac{\partial g}{\partial t_I} = 0 \quad (\text{A.18})$$

Hence, all the arguments used in the proof continue to apply after replacing the observed wage w_I by the shadow wage w_I^s . In particular, the first-order condition of the childcare-cost problem becomes:

$$w_I^s \cdot \nu \cdot \theta_I + \frac{\partial g}{\partial t_I} = 0.$$

To preserve the equivalence with the original resource constraint, the use of shadow wages must be accompanied by a corresponding adjusted full income. Define: $Y^s = Y + T \cdot (w_I^s - w_I)$ so that the decentralized problems retain the same form after replacing w_I by w_I^s and Y by Y^s . $\square$

Proof of Proposition 2.

The proof follows in two stages.

Part 1. Suppose first that (w_W, w_H, Y, n) is such that $m^*(w_W, w_H, Y, n) > 0$. Given that the childcare constraint is binding at equilibrium, we have:

$$m^*(w_W, w_H, Y, n) = g(t_W^*(w_W, w_H, Y, n), t_H^*(w_W, w_H, Y, n), n), \quad (\text{A.19})$$

Consider a neighborhood of (w_W, w_H, Y, n) in which $m^*(w_W, w_H, Y, n) > 0$. Differentiating both sides of this equation with respect to w_W and w_H , holding Y and n fixed, gives:

$$\frac{\partial m^*}{\partial w_W} = \frac{\partial g}{\partial t_W} \cdot \frac{\partial t_W^*}{\partial w_W} + \frac{\partial g}{\partial t_H} \cdot \frac{\partial t_H^*}{\partial w_W}, \quad (\text{A.20})$$

$$\frac{\partial m^*}{\partial w_H} = \frac{\partial g}{\partial t_W} \cdot \frac{\partial t_W^*}{\partial w_H} + \frac{\partial g}{\partial t_H} \cdot \frac{\partial t_H^*}{\partial w_H}. \quad (\text{A.21})$$

Equivalently,

$$\nabla m = \mathbf{J}_t \cdot \nabla g$$

where

$$\nabla m = \begin{pmatrix} \frac{\partial m^*}{\partial w_W} \\ \frac{\partial m^*}{\partial w_H} \end{pmatrix}, \quad \nabla g = \begin{pmatrix} \frac{\partial g}{\partial t_W} \\ \frac{\partial g}{\partial t_H} \end{pmatrix}, \quad \mathbf{J}_t = \begin{pmatrix} \frac{\partial t_W^*}{\partial w_W} & \frac{\partial t_H^*}{\partial w_W} \\ \frac{\partial t_W^*}{\partial w_H} & \frac{\partial t_H^*}{\partial w_H} \end{pmatrix}. \quad (\text{A.22})$$

Provided that the Jacobian matrix $\mathbf{J}_t$ is non-singular, $\det \mathbf{J}_t \neq 0$, this system of equations can be uniquely solved with respect to $\partial g / \partial t_W$ and $\partial g / \partial t_H$. Hence, we obtain:

$$\begin{aligned} \frac{\partial g}{\partial t_W} &= a(w_W, w_H, Y, n) \\ \frac{\partial g}{\partial t_H} &= b(w_W, w_H, Y, n) \end{aligned}$$

where $a(w_W, w_H, Y, n)$ and $b(w_W, w_H, Y, n)$ are identified functions. To interpret this rank condition, recall that $t_W^* = \zeta_W(\omega_W^*, \omega_H^*, n)$ and $t_H^* = \zeta_H(\omega_W^*, \omega_H^*, n)$. By the chain rule, $\mathbf{J}_t = \mathbf{J}_\omega \cdot \mathbf{J}_\zeta$, where

$$\mathbf{J}_\omega = \begin{pmatrix} \frac{\partial \omega_W^*}{\partial w_W} & \frac{\partial \omega_H^*}{\partial w_W} \\ \frac{\partial \omega_W^*}{\partial w_H} & \frac{\partial \omega_H^*}{\partial w_H} \end{pmatrix}, \quad \mathbf{J}_\zeta = \begin{pmatrix} \frac{\partial \zeta_W}{\partial \omega_W^*} & \frac{\partial \zeta_H}{\partial \omega_W^*} \\ \frac{\partial \zeta_W}{\partial \omega_H^*} & \frac{\partial \zeta_H}{\partial \omega_H^*} \end{pmatrix}.$$

Strict convexity of g implies that $\mathbf{J}_\zeta$ is non-singular. Consequently, $\det \mathbf{J}_\omega \neq 0$ implies that $\det \mathbf{J}_t \neq 0$. We make the following, rather weak assumption.

Invertibility condition. The matrix

$$\mathbf{J}_\omega = \begin{pmatrix} \frac{\partial \omega_W^*}{\partial w_W} & \frac{\partial \omega_H^*}{\partial w_W} \\ \frac{\partial \omega_W^*}{\partial w_H} & \frac{\partial \omega_H^*}{\partial w_H} \end{pmatrix}$$

is non-singular.

By local invertibility of the mapping from (w_H, w_W) into (t_W^*, t_H^*) , conditional on (Y, n) , the identified functions $a(w_W, w_H, Y, n)$ and $b(w_W, w_H, Y, n)$ can be expressed as functions of (t_W, t_H, n) . Hence, over the corresponding region of the support of (t_W, t_H) , the gradient $\nabla g(t_W, t_H, n)$ is identified. Since g is known to depend on (t_W, t_H, n) only, integration identifies $g(t_W, t_H, n)$ up to an additive function of n . The remaining function is pinned down by the boundary condition in (A.19). The prices of childcare time are then given by

$$\theta_W w_W = -\frac{\partial g}{\partial t_W}(t_W^*, t_H^*, n), \quad \theta_H w_H = -\frac{\partial g}{\partial t_H}(t_W^*, t_H^*, n)$$

from the first-order conditions of the cost minimization problem.

Part 2. Suppose now that (w_W, w_H, Y, n) is such that $m^*(w_W, w_H, Y, n) = 0$. In that case, the non-negativity constraint on external childcare expenditure is binding, and the optimal childcare times satisfy the boundary condition: $g(t_W^*(w_W, w_H, Y, n), t_H^*(w_W, w_H, Y, n), n) = 0$. We make the following assumption.

Support condition. Every relevant point of the boundary $g(t_W, t_H, n) = 0$ lies in the closure of the set of optimal parental childcare time allocations (t_W^*, t_H^*) associated with $m^* > 0$.

Here, the boundary refers to the boundary of the childcare technology in the (t_W, t_H) space, not to the boundary between the regimes $m^* > 0$ and $m^* = 0$ in the space of exogenous variables. Under the support condition, any relevant point on $g(t_W, t_H, n) = 0$ can be approached by a

sequence of optimal childcare allocations observed when $m^* > 0$. Since g is identified at these points, continuity identifies g at the limit. Under continuous differentiability, the same argument applies to its gradient, so that $\partial g/\partial t_W$ and $\partial g/\partial t_H$ are identified on the technological boundary as limits of their values in the region where $m^* > 0$.

The identification of the implicit prices of parental childcare time requires a further distinction. In the region where $m^* > 0$, the first-order conditions imply

$$\theta_W w_W = -\frac{\partial g}{\partial t_W}(t_W^*, t_H^*, n), \quad \theta_H w_H = -\frac{\partial g}{\partial t_H}(t_W^*, t_H^*, n)$$

so that both implicit prices are exactly identified. At a boundary point between the regions $m^* > 0$ and $m^* = 0$, continuity of the optimal choices and implicit prices extends this exact identification to the boundary. By contrast, at points lying strictly inside a region in which $m^* = 0$, the first-order conditions of Problem P_0 imply that

$$\nu \theta_W w_W = -\frac{\partial g}{\partial t_W}(t_W^*, t_H^*, n), \quad \nu \theta_H w_H = -\frac{\partial g}{\partial t_H}(t_W^*, t_H^*, n)$$

where $\nu \geq 1$ is a function of (w_W, w_H, Y, n) . Consequently, the two implicit prices are identified only up to a common positive multiplicative factor ν . This implies that:

$$\frac{\theta_W w_W}{\theta_H w_H} = \frac{\partial g/\partial t_W}{\partial g/\partial t_H}.$$

In other words, only the relative implicit price is identified. $\square$

Proof of Proposition 3. Consider first the individual problem P_I . Under the separability assumption, the first order conditions with respect to l_I , t_I and c_I , respectively, are:

$$\begin{aligned} \frac{\partial U_I}{\partial L_I}(L_I^*, c_I^*, c_K) &= \lambda_I w_I, \\ \frac{\partial U_I}{\partial L_I}(L_I^*, c_I^*, c_K) \cdot \varphi'_I(t_I^*) &= \lambda_I(1 - \theta_I)w_I, \\ \frac{\partial U_I}{\partial c_I}(L_I^*, c_I^*, c_K) &= \lambda_I, \end{aligned}$$

where λ_I is the Lagrange Multiplier of the spouse's budget constraint. Dividing the second condition by the first one gives: $\theta_I = 1 - \varphi'_I(t_I^*)$. Eliminating the redundant equation, the first order conditions become:

$$\begin{aligned}\frac{\partial U_I}{\partial L_I}(L_I^*, c_I^*, c_K) &= \lambda_I w_I, \\ \frac{\partial U_I}{\partial c_I}(L_I^*, c_I^*, c_K) &= \lambda_I,\end{aligned}$$

Using $l_I = L_I - \varphi_I(t_I^*)$, the individual budget constraint can be written as:

$$\kappa_I - c_I - L_I w_I + \varphi_I(t_I^*) w_I - t_I^*(1 - \theta_I) w_I \geq 0.$$

Define $\pi_I = \varphi_I(t_I^*) w_I - t_I^*(1 - \theta_I) w_I$. The budget constraint then becomes: $c_I + L_I w_I \leq \kappa_I + \pi_I$. Thus, conditional on κ_I , the remaining choice of L_I and c_I is equivalent to a standard consumer problem with wage w_I and virtual income $\kappa_I + \pi_I$. The first and third first-order conditions, together with this budget constraint, therefore determine L_I^* and c_I^* , with $I = W$ and H .

The first-order conditions of Problem P_0 are:

$$\nu \theta_W w_W + \frac{\partial g}{\partial t_W}(t_W^*, t_H^*, n) = 0, \tag{A.23}$$

$$\nu \theta_H w_H + \frac{\partial g}{\partial t_H}(t_W^*, t_H^*, n) = 0. \tag{A.24}$$

where $\nu \geq 1$ with $\nu = 1$ if $g(t_W^*, t_H^*, n) > 0$ and $\nu > 1$ if $g(t_W^*, t_H^*, n) = 0$. Substituting $\theta_I = 1 - \varphi'_I(t_I^*)$ in these equations, we obtain:

$$\nu(1 - \varphi'_W(t_W^*)) w_W + \frac{\partial g}{\partial t_W}(t_W^*, t_H^*, n) = 0, \tag{A.25}$$

$$\nu(1 - \varphi'_H(t_H^*)) w_H + \frac{\partial g}{\partial t_H}(t_W^*, t_H^*, n) = 0. \tag{A.26}$$

These two equations, using the condition on ν , can be solved with respect to t_W^* , t_H^* and ν , giving $\theta_W = 1 - \varphi'_W(t_W^*)$ and $\theta_H = 1 - \varphi'_H(t_H^*)$. Hence t_W^* , t_H^* and ν , and therefore θ_H and θ_W ,

are functions of w_W, w_H and n only. $\square$

Proof of Corollary 4. Under Assumption A.1 and Proposition 3, $\theta_I(t_I) = 1 - \varphi'_I(t_I)$, so that θ_I depends only on spouse I 's own childcare time. When $m^* > 0$, the first-order conditions imply

$$\theta_I(t_I^*) = -\frac{\partial g / \partial t_I}{w_I}.$$

Since the gradient of g is identified in this region, observations with $m^* > 0$ identify the function $\theta_I(\cdot)$ over the corresponding support of childcare time. The support condition implies that every relevant value of t_I arising on the technological boundary $g(t_W, t_H, n) = 0$ can be approached by values of t_I associated with $m^* > 0$. Hence, by continuity, $\theta_I(t_I)$ is identified at every such value. At observations with $m^* = 0$, the first-order conditions then give

$$\nu \theta_I(t_I^*) w_I = -\frac{\partial g}{\partial t_I},$$

so that

$$\nu = -\frac{\partial g / \partial t_I}{\theta_I(t_I^*) w_I}.$$

Thus, ν is identified as well. The expression can be computed using either spouse and must yield the same value of ν . $\square$

Appendix B: Additional Empirical Results

To assess the robustness of our results, we replicated the analysis using the 2017 wave of the PSID, which contains the same variables as the 2019 wave. The 2017 wave includes 9,607 households, to which we apply the same selection criteria as for the 2019 wave. Average values for unadjusted and adjusted childcare time in 2017 are reported in Table A.1. By construction, the average adjusted values are the same as in the 2019 wave. The average unadjusted values

differ only moderately, with fathers reporting about 2 fewer hours per week and mothers about 3.5 more hours. The descriptive statistics for the 2017 wave are reported in Table A.2. The estimated parameters of the childcare time and expenditure equations are reported in Table A.3. Overall, the estimates are somewhat less stable. To reduce the influence of extreme wage values, we exclude observations with an hourly wage rate below \$1 (4 observations) or above \$200 (2 observations). The resulting price-scale coefficients remain close to those obtained for 2019: in the benchmark specification (columns A), θ_W is around 0.66 and θ_H is around 0.18. As in the 2019 wave, the price-scale coefficient is somewhat larger for mothers than for fathers. For the other specifications, the estimates remain of a similar order of magnitude, except for the sample restricted to couples with positive childcare expenditure (columns D), for which the estimates are substantially less stable. In this specification, it was necessary to constrain the price-scale coefficients for the father to be equal to one. Finally, using the unadjusted measure of childcare time (columns E) yields slightly lower price-scale coefficients, as in the 2019 wave. Overall, the 2017 results support the conclusion that the θ_I parameters are below one.

Table A.1: Parents' Weekly Childcare Time (in Hours, with and without Adjustment) and Weekly Childcare Expenditure (in Dollars) for the 2017 Wave

	Mean	S.D.	Min	50%	Max	Pzeros
Father's Original PSID Childcare Time	31.95	36.23	0.00	20.00	168.00	0.01
Mother's Original PSID Childcare Time	57.14	48.85	0.00	40.00	168.00	0.01
Father's Adjusted Childcare Time	9.66	21.59	0.00	1.29	121.21	0.01
Mother's Adjusted Childcare Time	18.97	28.68	0.00	5.27	109.81	0.01
Childcare Expenditure	100.50	125.29	0.00	57.69	769.23	0.34

Notes: Mean = average value, S.D. = standard deviation, Min = minimum value, 50% = median, Max = maximum value, Pzeros = proportion of zeros.

Table A.2: Descriptive Statistics of the Benchmark Sample for the 2017 Wave

	Without Child		With Children	
	Mean	S.D.	Mean	S.D.
Father's Adjusted Childcare Time	0.00	0.00	9.66	21.59
Mother's Adjusted Childcare Time	0.00	0.00	18.97	28.68
Childcare Expenditure	0.00	0.00	100.50	125.29
No. of Children	0.00	0.00	1.59	0.49
Dummy for Child under 3	0.00	0.00	0.73	0.44
Husband's Leisure Time	125.16	12.78	115.74	22.40
Wife's Leisure Time	131.39	12.72	116.06	27.88
Husband's Hourly Wage	28.93	22.92	30.84	26.21
Wife's Hourly Wage	23.17	14.55	27.75	23.23
Full Expenditure (in thousand dollars)	5.26	2.97	5.90	3.38
Husband's Education in Years	14.24	2.20	14.23	2.34
Wife's Education in Years	14.83	2.19	15.01	2.09
Husband's Age in Years	36.31	7.60	34.16	5.22
Wife's Age in Years	35.12	7.62	32.69	4.86

Notes: S.D. = standard deviation.

Table A.3: ML Estimates of Childcare Time and Expenditure Equations for the 2017 Wave

		A		B		C		D		E	
Variable		Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
Childcare Expenditure Equation											
α_0	Intercept	-107.06	32.94	-86.32	36.88	-117.26	26.78	-42.16	30.13	-105.80	32.90
α_1	Husband's Educ.	-3.23	9.59	0.71	8.56	-2.03	7.79	1.48	7.85	-3.22	9.56
α_2	Wife's Educ.	14.78	9.66	10.19	8.67	17.52	7.89	-2.87	7.78	14.47	9.64
α_3	Husband's Age	-5.35	13.02	-2.78	12.06	-7.68	10.60	-4.81	10.25	-5.62	12.99
α_4	Wife's Age	26.76	13.30	21.72	12.35	19.35	10.78	11.02	10.40	26.82	13.27
α_5	Child under 3	-0.42	8.36	-4.82	7.65	-8.85	6.60	2.03	7.13	-0.34	8.33
α_6	No. of Children	26.33	8.14	16.92	7.42	-12.72	7.04	10.21	6.75	26.23	8.12
Father's Childcare Time Equation											
$\beta_{H,0}$	Intercept	-3.52	3.47	1.88	4.29	-4.65	2.82	4.33	0.95	13.89	5.81
$\beta_{H,1}$	Husband's Educ.	0.90	1.27	-0.24	1.24	0.24	1.07	0.67	1.10	1.77	2.13
$\beta_{H,2}$	Wife's Educ.	-0.48	1.29	-0.63	1.27	-0.31	1.09	1.05	1.11	-0.88	2.18
$\beta_{H,3}$	Husband's Age	0.33	1.75	0.29	1.78	0.16	1.47	1.09	1.47	-0.01	2.95
$\beta_{H,4}$	Wife's Age	-1.94	1.78	-1.37	1.82	-1.80	1.49	-0.90	1.46	-3.70	2.99
$\beta_{H,5}$	Child under 3	0.14	1.10	0.29	1.10	0.32	0.91	0.87	1.02	0.49	1.85
$\beta_{H,6}$	No. of Children	-0.09	1.07	-0.27	1.07	0.31	0.94	-0.91	0.97	-0.85	1.80
Mother's Childcare Time Equation											
$\beta_{W,0}$	Intercept	11.15	3.46	8.60	4.95	11.45	3.00	4.41	4.34	44.38	5.90
$\beta_{W,1}$	Husband's Educ.	2.22	1.63	1.30	1.68	0.58	1.33	1.51	1.61	4.21	2.79
$\beta_{W,2}$	Wife's Educ.	-2.99	1.69	-2.84	1.77	-2.03	1.38	-0.74	1.61	-4.16	2.88
$\beta_{W,3}$	Husband's Age	0.11	2.31	-0.11	2.48	1.31	1.88	0.65	2.12	0.34	3.95
$\beta_{W,4}$	Wife's Age	-1.58	2.34	-0.95	2.54	-2.61	1.90	-0.05	2.14	-3.20	4.00
$\beta_{W,5}$	Child under 3	0.88	1.45	1.42	1.54	2.56	1.16	0.23	1.47	1.07	2.47
$\beta_{W,6}$	No. of Children	-1.13	1.41	-1.27	1.50	-0.04	1.19	-0.07	1.39	-2.47	2.41
Wage Coefficients											
γ_H		26.97	7.78	15.18	9.06	28.12	6.20	17.03	0.06	31.49	9.84
γ_W		27.42	7.75	32.69	10.47	26.74	6.37	32.62	10.55	35.02	10.32
$\gamma_{W,H}$		-0.13	1.82	0.03	2.54	0.11	1.51	-1.10	0.04	-0.22	3.29
Price-Scale Coefficients											
θ_H		0.18	0.10	0.21	0.24	0.17	0.08	1.00	—	0.13	0.08
θ_W		0.66	0.33	0.52	0.29	0.65	0.27	0.39	0.48	0.40	0.21

Notes: Demographic variables are standardized. A = benchmark sample (420 observations), B = sample excluding extreme values (370), C = sample including couples with more than 2 children (618); D = sample excluding couples with zero childcare expenditure (273); E = sample using the original PSID childcare time measures (420).

References

- [1] Adda, Jérôme, Christian Dustmann, and Katrien Stevens(2017). “The Career Costs of Children”. *Journal of Political Economy*, 125 (2), 293-337
- [2] Apps, Patricia, and Ray Rees (2001). “Household production, full consumption and the costs of children.” *Labour Economics* 8 (6), 621-648.
- [3] Arduini, Francesca (2024). “Estimating intra-household sharing from time-use data”. IFS Working Paper. London: Institute for Fiscal Studies.
- [4] Bargain, Olivier, and Olivier Donni (2012). “Expenditure on children: A Rothbarth-type method consistent with scale economies and parents’ bargaining.” *European Economic Review* 56 (4), 792-813.
- [5] Bargain, Olivier, Olivier Donni, and Imen Hentati (2022). “Resource sharing in households with children: A generalized model and empirical evidence from the UK.” *Journal of the European Economic Association* 20 (6), 2468-2496.
- [6] Bargain, Olivier, Guy Lacroix, and Luca Tiberti (2022). “Intrahousehold Resource Allocation and Individual Poverty: Assessing Collective Model Predictions using Direct Evidence on Sharing.” *Economic Journal*, 132 (643), 865-905.
- [7] Bloemen, Hans, and Elena Stancanelli (2014). “Market hours, household work, child care, and wage rates of partners: an empirical analysis.” *Review of Economics of the Household*, 12 (1), 51-81.
- [8] Blundell, Richard, Pierre-André Chiappori, and Costas Meghir (2005). “Collective Labor Supply with Children.” *Journal of Political Economy* 113 (6): 1277–1306.
- [9] Blundell, Richard, and Thomas MaCurdy (1999). “Labor supply: A review of alternative approaches.” *Handbook of Labor Economics* 3, 1559-1695.
- [10] Bradbury, Bruce (2008). “Time and the cost of children.” *Review of Income and Wealth* 54 (3), 305-323.

- [11] Brown, Caitlin, Rossella Calvi, and Jacob Penglase (2021). “Sharing the pie: An analysis of undernutrition and individual consumption in Bangladesh.” *Journal of Public Economics* 200, 104460.
- [12] Budig, Michelle J., and Paula England (2001). “The wage penalty for motherhood.” *American Sociological Review* 66 (2), 204-225.
- [13] Calvi, Rossella, Jacob Penglase, Denni Tommasi, and Alexander Wolf (2023) “The more the poorer? Resource sharing and scale economies in large families.” *Journal of Development Economics*, 160, 102986.
- [14] Chiappori, Pierre-André (1988). “Rational household labor supply.” *Econometrica*, 56 (1), 63-90.
- [15] Chiappori, Pierre-André (1992). “Collective labor supply and welfare.” *Journal of Political Economy* 100 (3), 437-467.
- [16] Chiappori, Pierre-André, Bernard Fortin, and Guy Lacroix (2002). “Marriage market, divorce legislation, and household labor supply.” *Journal of Political Economy* 110 (1), 37-72.
- [17] Colombino, Ugo (2000). “The cost of children when children are a choice.” *Labour* 14 (1), 79-95.
- [18] Cosaert, Sam, and Veerle Hennebel (2023). “Parental childcare with process benefits.” *Economica* 90 (357), 339-371.
- [19] Cuevas-Ruiz, Pilar, José Ignacio Gimenez-Nadal, Sveva Manfredi, and Almudena Sevilla (2025). “A new model of parental time investments: a paradigm shift for addressing gender inequality in the labour market.” *Oxford Review of Economic Policy*, 41(3–4), 820–842.
- [20] Dechter, Evgenia Kogan (2014). “Maternity leave, effort allocation, and postmotherhood earnings.” *Journal of Human Capital* 8 (2), 97-125.
- [21] Diewert, W. E. (1971), “An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function.” *Journal of Political Economy*, 79 (3), 481-507.

- [22] Donni, Olivier (2007). “Choix collectifs des heures travaillées, consommation publique et équilibre de Lindahl.” *Revue économique*, 58 (3), 535-543.
- [23] Donni, Olivier (2009). “A simple approach to investigate intrahousehold allocation of private and public goods.” *The Review of Economics and Statistics* 91 (3), 617-628.
- [24] Donni, Olivier, and Pierre-André Chiappori (2011). “Nonunitary models of household behavior: a survey of the literature.” In: Molina, J. Alberto (ed.). *Household Economic Behaviors*, Springer New York, NY.
- [25] Donni, Olivier, and Eleonora Matteazzi (2018). “Collective decisions, household production, and labor force participation.” *Journal of Applied Econometrics* 33 (7), 1064-1080.
- [26] Dunbar, Geoffrey R., Arthur Lewbel, and Krishna Pendakur (2013). “Children’s resources in collective households: identification, estimation, and an application to child poverty in Malawi.” *American Economic Review* 103 (1), 438-471.
- [27] Glauber, Rebecca (2018). “Trends in the motherhood wage penalty and fatherhood wage premium for low, middle, and high earners.” *Demography* 55 (5), 1663-1680.
- [28] Gørtz, Mette (2011). “Home production—Enjoying the process or the product?” *Electronic International Journal of Time Use Research* 8 (1), 85-109.
- [29] Graham, John W., and Carole A. Green (1984). “Estimating the parameters of a household production function with joint products.” *The Review of Economics and Statistics*, 66 (2), 277-282.
- [30] Gronau, Reuben (1977). “Leisure, home production, and work—the theory of the allocation of time revisited.” *Journal of Political Economy* 85 (6), 1099-1123.
- [31] Gronau, Reuben (1991). “The Intrafamily Allocation of Goods—How to Separate the Adult from the Child.” *Journal of Labor Economics* 9 (3), 207-235.
- [32] Gustafsson, Björn, and Urban Kjulin (1994). “Time use in child care and housework and the total cost of children.” *Journal of Population Economics*, 7 (3), 287-306.

- [33] Hallberg, Daniel, and Anders Klevmarken (2003). "Time for children: A study of parent's time allocation." *Journal of Population Economics* 16 (2), 205-226.
- [34] Insolera, Noura E., David S. Johnson, and Beth A. Simmert (2019). "Evaluation of the Time Use data in PSID with comparisons to ATUS." *Panel Study of Income Dynamics, Technical Series Paper #19-02*.
- [35] Juster, F. Thomas, and Frank P. Stafford (1991). "The allocation of time: Empirical findings, behavioral models, and problems of measurement." *Journal of Economic literature*, 29 (2), 471-522.
- [36] Kalenkoski, Charlene M., David C. Ribar, and Leslie S. Stratton (2009). "The Influence of Wages on Parents' Allocations of Time to Child Care and Market Work in the United Kingdom." *Journal of Population Economics*, 22 (2), 399-419
- [37] Kerkhofs, Marcel, and Peter Kooreman (2003). "Identification and estimation of a class of household production models." *Journal of Applied Econometrics*, 18 (3), 337-369.
- [38] Kimmel, Jean, & Connelly, Rachel (2007). "Mothers' Time Choices: Caregiving, Leisure, Home Production, and Paid Work." *The Journal of Human Resources*, 42 (3), 643-681.
- [39] Korenman, Sanders, and David Neumark (1992). "Marriage, motherhood, and wages." *The Journal of Human Resources*, 27 (2), 233-255.
- [40] Koulovatianos, Christos, Carsten Schroder, and Ulrich Schmidt (2009). "Nonmarket household time and the cost of children." *Journal of Business & Economic Statistics* 27 (1), 42-51.
- [41] Lechene, Valérie, Krishna Pendakur, and Alex Wolf (2022). "Ordinary least squares estimation of the intrahousehold distribution of expenditure." *Journal of Political Economy*, 130 (3), 681-731.
- [42] Lewbel, Arthur, and Krishna Pendakur (2024). "Estimating a model of inefficient cooperation and consumption in collective households." *Review of Economics of the Household*, 22, 865-907.

- [43] Penglase, Jacob (2021). “Consumption inequality among children: Evidence from child fostering in Malawi.” *Economic Journal* 131 (634), 1000–1025.
- [44] Reid, Margaret G. (1934). “Economics of household production.” New York: John Wiley and Sons, Inc.
- [45] Stewart, Jay and Mary Dorinda Allard (2015). “Secondary Childcare in the ATUS: What Does It Measure?” IZA Discussion Paper No. 9376.
- [46] U.S. Bureau of Labor Statistics (2019). American Time Use Survey — 2018 Results. USDL-19-1003, June 19, 2019.
- [47] Waldfogel, Jane (1998). “The family gap for young women in the United States and Britain: can maternity leave make a difference?” *Journal of Labor Economics* 16 (3), 505-545.

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