Continuous Assessment in Applied Econometrics – 2 hours

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Detailed solutions – Mock exam 4

Answer the questions in order. All answers must be justified. Calculators are allowed. Throughout, log denotes the natural logarithm. Unless stated otherwise, the assumptions of the classical linear model, including normally distributed errors, are taken to hold. Critical values are read from the tables at the end of the paper. When the number of degrees of freedom is not printed, use the closest lower row, and use the row ∞ above 120. The marking scheme is indicative.

Exercise A – Training volume and marathon times ***(4 points)***

A1.

$$X'X = \begin{pmatrix} 10 & 50 \\ 50 & 286 \end{pmatrix} \qquad X'y = \begin{pmatrix} 2{,}670 \\ 12{,}963 \end{pmatrix}$$

$$\det(X'X) = 10 \times 286 - 50^2 = 360 \qquad (X'X)^{-1} = \frac{1}{360}\begin{pmatrix} 286 & -50 \\ -50 & 10 \end{pmatrix}$$

$$\hat{\beta} = \frac{1}{360}\begin{pmatrix} 286 \times 2{,}670 - 50 \times 12{,}963 \\ -50 \times 2{,}670 + 10 \times 12{,}963 \end{pmatrix} = \frac{1}{360}\begin{pmatrix} 115{,}470 \\ -3{,}870 \end{pmatrix} = \begin{pmatrix} 320.75 \\ -10.75 \end{pmatrix}$$

Ten more kilometres of weekly training are associated with a finishing time lower by 10.75 minutes ($\hat{\beta}_1 = -10.75$).

Marks: 0.5 $(X'X)^{-1}$ + 0.25 $\hat{\beta}$ + 0.25 interpretation in minutes per 10 km = 1

A2. $y = X\hat{\beta} + \hat{u}$, so $y'y = \hat{\beta}'X'X\hat{\beta} + 2\hat{\beta}'X'\hat{u} + \hat{u}'\hat{u}$. The middle term is zero ($X'\hat{u} = 0$) and $X'X\hat{\beta} = X'y$ (normal equations), hence $\hat{u}'\hat{u} = y'y - \hat{\beta}'X'y$. Numerically, $\hat{\beta}'X'y = 856{,}402.50 + (-139{,}352.25) = 717{,}050.25$ and SSR $= 717{,}804 - 717{,}050.25 = 753.75$.

Marks: 0.5 proof + 0.5 SSR = 1

A3. SST $= 717{,}804 - 2{,}670^2/10 = 4{,}914$, so $R^2 = 1 - 753.75/4{,}914 = 0.8466$ and $\hat{\sigma}^2 = 753.75/(10-2) = 94.22$ (divisor $N - K$). With a constant, $R^2 = \text{corr}(y, \hat{y})^2$ and the correlation is positive: $\text{corr}(y, \hat{y}) = 0.9201$.

Marks: 0.25 R^2 + 0.25 $\hat{\sigma}^2$ with $N - 2$ + 0.5 correlation with its justification = 1

A4. $\hat{\gamma} = \sum x_i y_i / \sum x_i^2 = 12{,}963/286 = 45.3252$ and, by the same identity, SSR $= \sum y_i^2 - \hat{\gamma}\sum x_i y_i = 130{,}253.8$, 173 times larger than with a constant. Then

$$1 - \frac{130{,}253.8}{4{,}914} = -25.51 \qquad 1 - \frac{130{,}253.8}{717{,}804} = 0.8185$$

The first (centred) number is hugely negative: the coach's line does far worse than simply predicting the average time (it predicts 226.6 minutes for a runner doing 50 km a week, against 267.0 with a constant). It can be negative because, without a constant, $\sum_i \hat{v}_i \neq 0$ and SST $\neq$ SSE + SSR. The second (uncentred) number looks respectable only because its benchmark is a prediction of zero minutes. The model is absurd: the relationship is only meant to hold over the observed range, and forcing it through the origin destroys the fit.

Marks: 0.25 $\hat{\gamma}$ and SSR + 0.25 the two numbers + 0.5 interpretation of each = 1

Typical mistakes.

- Inverting $X'X$ by inverting each entry.

- Dividing SSR by N or by $N-1$.
- Concluding from 0.82 that the coach's model fits "almost as well".

Exercise B – What fills cinema seats? ***(6 points)***

B1. The model is log-log. A 1% increase in the number of screens is associated with about 1.085% more admissions, for given weeks since release, sequel status and holidays (10% more screens: about 10.8% more admissions). The elasticity is above one, so admissions grow more than proportionally with screens.

Marks: 1

B2. From w to $w+1$, $\Delta y = \beta_w + \beta_{w^2}\,[(w+1)^2 - w^2]/10 = \beta_w + \beta_{w^2}(2w+1)/10$, and the exact change is $100(e^{\Delta y}-1)$.

$$\text{week 2 to 3}: -0.401 + 0.141 \times 0.5 = -0.331 \quad 100(e^{-0.331}-1) = -28.1\%$$
$$\text{week 8 to 9}: -0.401 + 0.141 \times 1.7 = -0.161 \quad 100(e^{-0.161}-1) = -14.9\%$$

Admissions fall every week, quickly at first (about 28% between weeks 2 and 3), then more slowly (about 15% between weeks 8 and 9). A film loses most of its audience early. The derivative $\beta_w + 2\beta_{w^2} w/10$ would vanish at $w = 14.2$, outside the observed 1–12 weeks.

Marks: 0.5 first value + 0.5 second value + 0.5 interpretation = 1.5

B3. The claim gives a direction, so $H_0 : \beta_{seq} = 0$ against $H_1 : \beta_{seq} > 0$. Then $t = 0.195/0.040 = 4.88 > t_{0.95}(1{,}194) \approx 1.645$ and we reject H_0. A sequel attracts $100(e^{0.195}-1) = 21.5\%$ more admissions, other things equal. The "original film" dummy equals one minus the sequel dummy, so the two dummies add up to the column of ones. The columns of X are linearly dependent, $X'X$ is singular and $\hat{\beta}$ does not exist (dummy variable trap). One category must remain the reference.

Marks: 0.5 one-sided test + 0.5 trap with $X'X$ = 1

B4. $H_0 : \beta_{seq} = \beta_{hol} = 0$ and column (2) is the restricted model. Both models have the same y and the same sample and are nested, so

$$F = \frac{(0.833 - 0.826)/2}{(1 - 0.833)/1{,}194} = \frac{0.0035}{0.000140} = 25.0 > F_{0.95}(2,\, 1{,}194) \approx 3.00$$

and we reject H_0. With R^2s rounded to three decimals the value is approximate.

Marks: 0.5 statistic + 0.5 decision = 1

B5. The omitted variable identity is $\tilde{\beta}_{ls} = \hat{\beta}_{ls} + \hat{\beta}_{lm}\hat{\delta}$, that is $0.816 + 0.292 \times 0.922 = 0.816 + 0.269 = 1.085$, the value of column (3). Films with large marketing budgets are released on more screens ($\hat{\delta} > 0$), and marketing raises admissions ($\hat{\beta}_{lm} > 0$). Column (3) therefore attributes part of the marketing effect to screens (positive bias). The screen elasticity falls below one once marketing is held constant.

Marks: 0.5 identity + 0.5 economic explanation = 1

B6. Heteroskedasticity leaves the estimates unbiased but makes the usual standard errors, t and F invalid. With $t = 4.88$ and $F = 25.0$ far from the critical values, the conclusions are unlikely to change, but standard errors valid under heteroskedasticity should be used.

Marks: 0.5

Typical mistakes.

- Reading $\hat{\beta}_{ls}$ as "one more screen raises admissions by 108%".
- Interpreting $\hat{\beta}_w$ alone as the weekly decline, or using the derivative instead of the discrete change from w to $w+1$.
- Two-sided test in B3.

Exercise C – Do hospital costs have economies of scale? ***(5 points)***

C1. $H_0 : \beta_2 + \beta_3 = 1$. Multiplying all input prices by λ multiplies the cost of producing a given output by λ. When $\log w$ and $\log p_E$ both increase by $\log\lambda$, $\log C$ increases by $(\beta_2+\beta_3)\log\lambda$, which must equal $\log\lambda$.

Marks: 0.5 hypothesis + 0.5 explanation = 1

C2. $\hat\beta_2 + \hat\beta_3 = 0.925$ and

$$\widehat{\mathrm{Var}}(\hat\beta_2+\hat\beta_3) = 0.005041 + 0.002704 + 2(-0.0021) = 0.003545 \qquad \text{s.e.} = 0.0595$$

$$t = \frac{0.925 - 1}{0.0595} = -1.26$$

N is unknown, so the degrees of freedom $N-4$ are unknown. But $t = -1.26$ is below 1.96 in absolute value, and 1.96 is the smallest two-sided 5% critical value in Table G.2 (row ∞), all the others being larger. Homogeneity is therefore not rejected, whatever N. Without the covariance we would get s.e. 0.0880 and $t = -0.85$, the same decision here, but by luck.

Marks: 0.75 variance with +2 Cov + 0.25 t + 0.5 argument on N = 1.5

C3. Substituting $\beta_3 = 1-\beta_2$ gives $\log C = \beta_0 + \beta_1\log Q + \beta_2\log w + (1-\beta_2)\log p_E + u$, that is

$$\log(C/p_E) = \beta_0 + \beta_1\log Q + \beta_2\log(w/p_E) + u$$

Regress the log of the cost deflated by the energy price on a constant, $\log Q$ and the log relative wage. β_3 is recovered as $1-\hat\beta_2$.

Marks: 0.5 substitution + 0.5 model with deflated variables = 1

C4. A 10% increase in stays raises costs by about 9.1% (exactly $100(1.1^{0.912}-1) = 9.08\%$). Economies of scale mean that costs rise less than proportionally with output, so $H_0 : \beta_1 = 1$ against $H_1 : \beta_1 < 1$. Then $t = (0.912-1)/0.038 = -2.32$, and we reject when $t < -t_{0.95}(N-4)$. In Table G.2, $t_{0.95}(3) = 2.353$ (do not reject) and $t_{0.95}(4) = 2.132$ (reject), and the critical value keeps decreasing as the degrees of freedom increase. The conclusion "economies of scale" therefore holds as soon as $N-4 \geq 4$, that is $N \geq 8$ hospitals. A national sample is certainly larger.

Marks: 0.5 interpretation + 0.25 one-sided hypothesis + 0.25 t + 0.5 threshold on N = 1.5

Typical mistakes.

- Writing $H_0 : \beta_2 + \beta_3 = 0$ (that is homogeneity of degree zero).
- Dividing C by w *and* by p_E in C3.
- Using a two-sided test in C4 (threshold $N \geq 12$).

Exercise D – How much concentrate should a dairy cow get? ***(5 points)***

D1. By Frisch–Waugh–Lovell, $\hat\beta_3 = 193.0/44.53 = 4.334$, with standard error $\hat\sigma/\sqrt{\sum \hat r_i^2} = 2.537/6.673 = 0.380$ ($t = 11.4$). For a given feed ration, Holstein herds produce about 4.3 more litres per cow per day.

Marks: 0.5 coefficient + 0.5 standard error + 0.5 interpretation = 1.5

D2. $\tilde\beta_1 = \hat\beta_1 + \hat\beta_3\hat\delta_1 = 2.450 + 4.334 \times 0.1241 = 2.450 + 0.538 = 2.988$. This is the coefficient of f in the short regression. Holstein herds receive more concentrate ($\hat\delta_1 > 0$) and produce more for a given ration ($\hat\beta_3 > 0$). Omitting the breed attributes part of the breed effect to feed (positive bias).

Marks: 0.5 identity + 0.5 direction = 1

D3. The marginal product is $\beta_1 + 2\beta_2 f$. Setting it to 0.80 gives $f^* = (0.80-\beta_1)/(2\beta_2)$, so

$$\hat f^* = \frac{0.80 - 2.450}{2 \times (-0.1022)} = 8.07 \text{ kg}$$

Jacobian: $\partial f^*/\partial \beta_1 = -1/(2\beta_2) = 4.892$, $\partial f^*/\partial \beta_2 = -(0.80 - \beta_1)/(2\beta_2^2) = 79.0$. With $\widehat{\mathrm{Var}}(\hat{\beta}_1) = 0.218089$, $\widehat{\mathrm{Var}}(\hat{\beta}_2) = 0.00108241$:

$$\widehat{\mathrm{Var}}(\hat{f}^*) = 5.2200 + 6.7530 - 11.6006 = 0.3724 \qquad \text{s.e.} = 0.610 \text{ kg}$$

The 95% interval is $[6.88,\ 9.27]$ kg. 10 kg lies outside ($t = -3.16$), so the recommendation is too generous. The data support about 7 to 9 kg. The covariance is essential. Without it, the standard error would be 3.460 kg and 10 kg would look compatible.

Marks: 0.5 $\hat{f}^*$ + 0.5 Jacobian + 0.5 variance with the covariance + 0.5 interval and verdict = 2

D4. Management is an omitted variable positively correlated with feed and with yield. $\hat{\beta}_1$ (and possibly $\hat{\beta}_2$) captures part of the management effect, so the marginal product of feed is overstated, and so is f^*. The true optimum is probably lower than 8.07 kg.

Marks: 0.5

Typical mistakes.

- In D1, using the $\hat{\sigma}$ of a short regression or dividing by $\sum h_i^2$.
- In D3, setting the marginal product to zero (that maximises milk, not profit), or ignoring the covariance.