The Household Demand for Leisure, the Price of Time and the Full Cost of Children: A Structural Model and Evidence from the PSID *

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Abstract

We propose a novel approach to estimating the full cost of children — the sum of monetary and time costs — by endogenizing the price of parental time rather than assuming it is equal to the parents' wage rate. In this approach, the price of time depends on how parents perceive their time with children, whether as a leisure-like activity or more as a labor-like activity. We then develop a simplified collective model of leisure demand for working couples, incorporating individual preferences and childcare technology, and estimate it using 2019 PSID data. This allows us to recover the price of parental time and the full cost of children. We find that mothers perceive 44% of their childcare time as labor, compared to 35% for fathers. Our results also highlight that a substantial portion of the full cost of children is non-monetary.

JEL Classification: I30, I31, J21, J22, J31

Keywords: Labor Supply, Leisure Demand, Collective Model, Price of time, Identification, Resource sharing, Cost of children.

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1 Introduction

The cost of children is a critical parameter used to implement many economic policies or to calculate inequality measures. Economists have long developed methods to infer from survey data what parents spend for children (see Bargain and Donni, 2012; Bargain, Donni and Hentati, 2022; Bargain, Lacroix, and Tiberti, 2022; Brown, Calvi, and Penglase, 2021; Calvi et al., 2023; Dunbar, Lewbel and Pendakur, 2013; Lechene, Pendakur, and Wolf, 2022; Lewbel and Pendakur, 2024; Penglase, 2021, for recent applications). However, focusing solely on monetary expenditures provides an incomplete picture of the full cost of children. The full cost of children indeed consists of both a monetary cost — the purchase of goods and services that contribute to children's well-being — and a time cost, which reflects the value of time parents and other caregivers dedicate to children.[1]

Evaluating the time cost is particularly complex and has been largely overlooked, with only a few exceptions. Gustafsson and Kjulin (1994) value parents' time by using either wage rates or the price of equivalent services. They then combine this with survey-based measures of monetary expenditures to construct a measure of the full cost of children. Apps and Rees (2001) develop a structural model to estimate the full cost of children, incorporating both monetary and time costs. Their model assumes linear homogeneity in childcare technology, identical parental preferences, and that the price of time is determined by the parents' wage rates. Similarly, Colombino (2000) constructs a simple structural model where parental time is also valued at the wage rate.[2] Finally, Bradbury (2008) presents a theoretical framework and numerical illustrations, where the full cost of children is inferred from variations in parents' leisure, following a Rothbarth-style approach.[3]

One of the main challenges in evaluating the full cost of children is assigning a value to childcare

[1] To be comprehensive, other long-term costs may also arise, such as reduced career advancement opportunities (see Adda, Dustmann, and Stevens, 2017; Budig and England 2001; Dechter, 2014; Glauber, 2018; Korenman and Neumark, 1992; Waldfogel, 1998).

[2] In this model, the number of children is explicitly treated as a household decision.

[3] Koulovatianos, Schröder and Schmidt (2009) adopt a different identification strategy, recovering the full cost of children from subjective questions rather than observed parental behavior.

time. Typically, the price of time a spouse dedicates to any productive activity is assumed to align with their wage rate. However, this approach relies on two strong hypotheses: (a) the working time of the spouse has to be freely chosen, without any constraints such as non-participation in the labor market, and (b) the time dedicated to the activity is perfectly substitutable to market working time, with a marginal rate of substitution equal to one. If these conditions do not hold, the price of childcare time becomes endogenously determined, depending on parental preferences and childcare technology. The second assumption is particularly problematic in the childcare context. For example, Cosaert and Hennebel (2023) show that a large fraction of childcare time is perceived as leisure by parents. Juster and Stafford (1991) formalize this idea through the concept of 'process benefits', which captures how certain activities generate direct utility for individuals beyond its intended outcome.

The aim of this paper is to evaluate the full cost associated with raising children, using a structural framework capable of delineating the price of parents' childcare time. To achieve this, we develop a collective labor supply model for a working couple with children. Each parent is assumed to have a distinct utility function that depends on his or her leisure time, childcare time, and consumption with the crucial feature that childcare time is imperfectly substitutable for working time. More generally, childcare time can be viewed for parents as either a leisure-like activity that increases utility — reflecting the concept of process benefits — or a labor-like activity that does not affect it or even decreases it.[4] In addition to influencing utility, childcare time is also integrated into a childcare technology alongside other monetary inputs — what we call a 'pure' childcare activity. Only this pure childcare activity can be seen as representing a cost for parents. Consequently, the price of parental time devoted to pure childcare is not equal to wage rate; instead, it is determined by the substitutability rate between childcare time and the external childcare services purchased on the market. To complete the model, we assume that parents incur expenses for their children according to a predetermined rule. The full cost

[4]To be clear, market work time does not enter the utility function in our specification; it serves as the reference use of time. Accordingly, if childcare time is perfectly substitutable for market work, it has no direct effect on utility. Note also that the work–leisure distinction is not binary but continuous: a unit of childcare time may yield more utility than leisure time, less than market work time, or anything in between.

of children is finally assumed to comprise both the value of parental time devoted to the pure childcare activity (and not its effect on parents' utility) and the direct expenses for children.

Our main result is that the full cost of children can be identified from observed behavior. Specifically, our model allows us to break down each unit of childcare time into a pure childcare activity and a leisure-like (or labor-like) activity that directly affects the parents' utility. We then estimate this model using a sample of dual-earner couples, both with and without children, from the 2019 wave of the Panel Study of Income Dynamics (PSID). This dataset offers detailed information on time allocation, expenditures (including external childcare services), and socio-demographic variables. Our empirical analysis then proceeds in two stages. First, we estimate the childcare technology and determine the price of pure childcare time, enabling us to calculate the time cost of children and define a composite measure of full leisure that includes the leisure-like (or labor-like) component of childcare activities. Second, we estimate full leisure demand functions to derive the full cost of raising children in a Rothbarth-like approach. Our estimates indicate that, depending on the specification, 32–66% of mothers' childcare tasks are effectively pure childcare time, compared with 20–42% for fathers, implying the price of pure childcare time is substantially below market wages. For a dual-earner couple with one child, the average time cost of raising children is estimated at approximately \$430–\$810 per week while the monetary cost at approximately \$720–\$740 per week. However, the monetary cost estimates are more fragile, for reasons discussed below.

Using a theoretically consistent framework, we show that market wages are generally inappropriate for valuing the time cost of children. On that basis, we are — to our knowledge — the first to identify shadow prices for pure childcare time and to estimate the time cost of children from those prices. This approach contrasts with prior studies, which either rely on wages to value time or focus solely on consumption data, addressing only the monetary component of the cost of children. Our procedure then recovers the full cost of children (the monetary cost is obtained residually as the difference between the full cost and the time cost). While somewhat similar to Rothbarth-like approaches — which compare adult consumption between couples with and without children — our method to recover the full cost of children introduces several key innovations. The 'adult good' we analyze is parents' full leisure, which we define to include a portion of childcare time, with this fraction determined by a childcare technology function. Moreover, the demand equation is conditioned on full expenditure, i.e., total expenditure inclusive of the opportunity cost of leisure, rather than on conventional total expenditure. Our approach thus differs from approaches which focus on consumption goods rather than time use.[5] Moreover, unlike Bargain and Donni (2012) and Bargain, Donni, and Hentati (2022), we do not rely on data from single individuals and, unlike Dunbar, Lewbel, and Pendakur (2013), we refrain from imposing strong parametric restrictions on inter-individual preferences.[6] Conversely, our contribution can be related to the few studies that have attempted to estimate process benefits, though not specifically focused on childcare. Graham and Green (1984) were pioneers in this area, yet they did not address issues related to identification.[7] Kerkhofs and Kooreman (2003) conducted a thorough examination of the identification problems associated with process benefits, ultimately reaching rather pessimistic conclusions, while also providing empirical results. Cosaert and Hennebel (2023), using a nonparametric approach, appear to be the only ones who explicitly consider process benefits in the context of childcare.[8]

The rest of this paper is as follows. Section 2 presents the theoretical model. Section 3 outlines the empirical specification and the data. Section 4 describes the estimation method and the empirical results. Finally, section 5 concludes.

[5]Our approach also differs from that of Arduini (2024), who relies on measures of leisure time but does not focus on children's cost.

[6]Nonetheless, consistent with this study, we impose the Similarity Across Types (SAT) condition.

[7]Graham and Green (1984) use a Cobb-Douglas specification for the household production function in two-adult households. They also introduced a specification for the jointness functions, enabling an analytical solution to the household optimization problem. See also Gørtz (2011) for an application with Danish data.

[8]Hallberg and Klevmarken (2003) use direct survey questions to elicit the process benefits associated with time spent on childcare.

2 The Model

2.1 Decision-making in a couple with children

We first consider a two-adult household, consisting of a wife (W) and a husband (H), with their n children, who make decisions about leisure, childcare, and consumption in a static framework.[9] The wife's and husband's leisure time and consumption are respectively denoted by l_W, l_H, c_W and c_H while their childcare time are respectively denoted by t_W and t_H.[10] Spouses have specific preferences for how they allocate their time and consumption. More precisely, each spouse I, with $I = W,\ H$, has a utility function of the form:

$$u_I = u_I(l_I, t_I, c_I, n) + \delta_I(n) \cdot u_K(c_K),$$

where u_I is a differentiable function, strictly increasing in l_I and c_I, and strongly concave in l_I, t_I and c_I, while u_K is a differentiable function, strictly increasing and strongly concave in c_K.

This formulation assumes that parents are altruistic toward their representative child, whose utility is captured by $u_K(c_K)$, where c_K is the child's consumption of goods and services. The degree of altruism of each parent is represented by $\delta_I(n)$, with $\delta_I(0) = 0$.[11] For simplicity, we consider only parent-to-child altruism, but at the cost of additional notational complexity, the model could be extended to account for mutual altruism between parents, as discussed by Donni and Chiappori (2011). It is also noteworthy that childcare time directly enters parents' utility functions, acting more as a leisure-like activity if $\partial u_I / \partial t_I > 0$ or a labor-like activity if $\partial u_I / \partial t_I \leq 0$ and capturing the aforementioned process benefits.[12] Finally, we assume that

[9] The terms 'wife' and 'husband' are used here for simplicity, and the partners are not necessarily married.

[10] What we define as leisure time may include certain productive activities, as long as they do not involve childcare time. Donni (2008) shows that this extension is straightforward under the assumption that the household production function is additively separable in spouses' time inputs.

[11] The number of children n can also be understood, mutatis mutandis, as a vector including all the characteristics of the children, and not only their number.

[12] This feature generalizes the model proposed by Donni and Vil (2024). The few labor supply models that incorporate domestic production generally assume perfect substitutability between non-market working time and

$\lim_{c_I \to 0} \partial u_I / \partial c_I = \lim_{c_K \to 0} \partial u_K / \partial c_K = +\infty$, which ensures that consumption is always strictly positive in equilibrium; and $\lim_{l_I \to 0} \partial u_I / \partial l_I = +\infty$ with $\partial u_I / \partial t_I < \partial u_I / \partial l_I$, so that parents always value childcare time less than leisure. This condition guarantees us that leisure time is always positive.

To simplify the intuition, we do not assume that childcare time directly enters the child's utility function.[13] Instead, we impose a childcare constraint, given by:

$$m - g\left(t_W, t_H, n\right) \geq 0, \tag{1}$$

where m is the money dedicated to purchase external childcare services and g is a differentiable function, strictly decreasing and strictly convex in its first two arguments — which guarantees that isoquants between m and t_W or t_H are decreasing and convex with respect to the origin — and decreasing in its last argument. The childcare constraint can be viewed as a technological constraint that must be fulfilled to ensure that children receive supervision round the clock. Additionally, we assume that

$$m \geq 0, \tag{2}$$

which ensures that external childcare expenditures can be zero or positive but never negative. For simplicity, we rule out the possibility of zero childcare time, by assuming $\lim_{t_W \to 0} \partial g / \partial t_W = \lim_{t_H \to 0} \partial g / \partial t_H = -\infty$. Finally, one limitation should be noted. This specification does not explicitly account for the possibility of parents jointly caring for children, nor does it consider the potential increased profitability for children that may result from joint care.

At this stage, it is important to clarify that we define childcare time strictly as the time actively

market working time (Gronau, 1977; Donni, 2008). This assumption is particularly strong and often unrealistic when applied specifically to childcare time rather than general non-market working time.

[13]The main theoretical results of this paper still hold if the childcare constraint that follows is replaced by the assumption that the child's utility function is separable in t_W, t_H and m, taking the form $u_K(c_K, f_K(t_W, t_H, m, n))$ for some well-behaved function $f_K(\cdot)$. The crucial assumption here is separability, which allows the time cost of children to be isolated from other costs.

dedicated to caring for children — time that is explicitly taken away from either paid work or leisure. In other words, childcare requires the full attention of a parent and cannot be performed concurrently with other activities.[14] Under this assumption, the household traditional budget constraint — with the prices of both parents' and children's consumption goods normalized to one — is of the form:

$$Y - c_W - c_H - c_K - l_W w_W - l_H w_H - t_W w_W - t_H w_H - m \geq 0 \tag{3}$$

where $Y = T \cdot (w_W + w_H) + y$ is the household full income, T the total time endowment of each spouse, y other nonlabor income, and w_W and w_H the spouses' wage rates.

The decision process is assumed to lead to Pareto efficient outcomes. For a couple with children, the optimization problem of the household is thus:

$$\max_{l_W,l_H,t_W,t_H,c_W,c_H,c_K,m} \phi_W u_W(l_W, t_W, c_W, n) + \phi_H u_H(l_H, t_H, c_H, n) + (\phi_W \delta_W + \phi_H \delta_H) u_K(c_K) \tag{$\bar{\text{P}}$}$$

with $\phi_W + \phi_H = 1$, subject to the childcare constraint (1), the non-negativity constraint (2), the budget constraint (3) and time feasibility conditions: $T - l_W - t_W \geq 0$ and $T - l_H - t_H \geq 0$. In this problem, ϕ_W and ϕ_H are Pareto weights that may generally depend on all the exogenous variables. They determine the location along the Pareto frontier. If $\phi_W = 0$ (or $\phi_H = 1$), then the household behaves as though the husband always gets his way, whereas, if $\phi_W = 1$ (or $\phi_H = 0$), it is as if the wife is the effective dictator. If Pareto weights are constant, the optimization problem simplifies to the maximization of a separable household utility function, consistent with the unitary approach. If Pareto weights are functions of wage rates and other exogenous incomes — reflecting the idea that these variables are indicators of bargaining power[15] — then the optimization problem corresponds to the collective approach stricto sensu. Finally,

[14] This contrasts with broader definitions that include passive supervision — such as simply being present while performing other activities. As we will see, the PSID adopts this more inclusive approach to measuring childcare time, which introduces challenges for empirical implementation within our framework.

[15] The bargaining weight may also be a function of distribution factors, i.e., variables that influence bargaining power without affecting the budget constraint.

the expenditure allocated to the child reflects the interaction between parents' altruism and their relative bargaining power.

As is common with collective models (see Donni and Chiappori, 2011), the optimization problem can be decentralized. If we assume that the time feasibility conditions are not binding at the optimum (implying that market work time is strictly positive), we have the following result.

Proposition 1. *Assume that time feasibility conditions are not binding. Then the optimal allocation of time and consumption in Problem $\bar{P}$ can be seen as the solution of a decentralized decision process. More precisely,*

(a) *there exists a pair of positive functions (θ_W, θ_H) of (w_W, w_H, Y, n) such that the time cost is minimized:*

$$e_T(w_W, w_H, Y, n) = \min_{t_W, t_H} \left(t_W \theta_W w_W + t_H \theta_H w_H + g\left(t_W, t_H, n\right)\right); \tag{P$_0$}$$

(b) *there exists a triplet of functions $(\kappa_W, \kappa_H, \kappa_K)$ of (w_W, w_H, Y, n), with $\kappa_W + \kappa_H + \kappa_K = Y - e_T$, such that each spouse maximizes her or his own utility function subject to a budget constraint:*

$$\max_{l_W, t_W, c_W} u_W(l_W, t_W, c_W, n) \text{ subject to } \kappa_W - c_W - l_W w_W - t_W(1-\theta_W)w_W \geq 0, \tag{P$_H$}$$

$$\max_{l_H, t_H, c_H} u_H(l_H, t_H, c_H, n) \text{ subject to } \kappa_H - c_H - l_H w_H - t_H(1-\theta_H)w_H \geq 0. \tag{P$_W$}$$

The proofs are in Appendix A. Focusing on interior solutions for market work time is important because it ensures that wage rates are observed and represent a convenient measure of the price of time. Intuitively, the spouses first minimize the time cost required to satisfy the childcare constraint, with $\theta_I w_I$ as the price of pure childcare time. They then allocate their remaining resources between themselves and their child according to the functions κ_W, κ_H and κ_K and maximize the utility functions u_W and u_H, with $(1-\theta_I)w_I$ as the price of leisure-like childcare time. Each unit of childcare time by spouse I is therefore valued at $\theta_I w_I$ as a pure childcare activity and at $(1-\theta_I)w_I$ as a leisure-like activity if $\theta_I < 1$ (or as a labor-like activity if $\theta_I \geq 1$). If $\theta_I = 1$, there is no process benefits; similar to market labor time, childcare time simply reduces parents' leisure time.[16]

The price of pure childcare time $w_I^* = \theta_I w_I$ and the price of leisure-like time $\bar{w}_I^* = (1-\theta_I)w_I$ are similar to Lindahl prices found in public economics,[17] and their sum equals w_I. The time cost of children, defined as $e_T(w_W, w_H, Y, n)$, is evaluated using the prices $w_W^* = \theta_W w_W$ and $w_H^* = \theta_H w_H$, and not the traditional wage rates. If childcare time is perceived as a leisure-like activity, meaning that w_I^* is lower than w_I, then — ceteris paribus — the time cost of children is overestimated when evaluated using standard wage rates instead of the correct prices. If w_I^* is greater than w_I, then it is underestimated. The full cost is defined as the sum of the time cost $e_T(w_W, w_H, Y, n)$ and the monetary cost $e_M(w_W, w_H, Y, n)$, the latter being equal to $\kappa_K(w_W, w_H, Y, n)$.

Finally, it is important to note that this result holds even when the constraint (2) is binding, i.e., when the household does not purchase external childcare services.

2.2 An Empirically Tractable Model

The primary objective of this study is to show how the full cost of children — and its decomposition into time cost and monetary cost — can be identified from traditional survey data. To do so, we adopt a few simplifying assumptions.

2.2.1 The Full Leisure Demand Functions

The decision process is not strictly speaking two-staged or sequential as the first stage Problem P_0 depends on the prices of pure childcare time, which are also determined by the second stage Problems P_W and P_H. To obtain a clear two-stage decision framework, we adopt the

[16]The case where $\theta_I \leqslant 0$ is excluded by the assumption $(\partial u_I/\partial t_I)/(\partial u_I/\partial l_I) < 1$. Without this assumption, the analysis would be significantly more complicated.

[17]See also Donni (2007, 2009) for an application to the household.

assumption used by Green and Graham (1984) and Donni and Matteazzi (2018), imposing additional structure on the utility functions as described below.

Assumption A.1 Each spouse I has a utility function of the form:

$$u_I(l_I, t_I, c_I, n) = u_I(l_I + \varphi_I(t_I), c_I, n)$$

with $I = W$ or H, where $L_I = l_I + \varphi_I(t_I)$ can be viewed as the 'full leisure' of spouse I for some differentiable and concave functions $\varphi_I(\cdot)$, satisfying $\varphi_I(0) = 0$.

This additional structure preserves the core properties of our model while making it more tractable for empirical estimation. It offers two main advantages. Firstly, leisure time and leisure-like childcare time are combined into a single composite full leisure time. This definition can be applied consistently to both couples with and without children, facilitating comparisons between them. Secondly, the decision process is a two-staged one. Specifically, the price of the leisure-like activity in the second stage Problem P_I is determined by the first-stage choice of childcare time only, as detailed in the following proposition.

Proposition 2. *Assume that time feasibility conditions are not binding and that utility functions are of the form A.1. Then the optimal allocation of time and consumption in Problem* $\bar{P}$ *is sequential. Firstly, the time cost is minimized in Problem* P_0 *with* $\theta_W = 1 - \varphi'_W(t_W)$ *and* $\theta_H = 1 - \varphi'_H(t_H)$*, giving* $t^*_H = t_H(w_W, w_H, n)$ *and* $t^*_W = t_W(w_W, w_H, n)$ *as optimal levels of childcare time. Secondly, each spouse maximizes her or his own utility function subject to a budget constraint in Problems* P_W *and* P_H *with* $\theta_W = 1 - \varphi'_W(t^*_W)$ *and* $\theta_H = 1 - \varphi'_H(t^*_H)$*, giving* $L^*_W = L_W(w_W, \kappa_W + \pi_W)$ *and* $L^*_H = L_H(w_H, \kappa_H + \pi_H)$ *as optimal levels of full leisure, with* $\pi_W = \varphi_W(t^*_W)w_W - t^*_W(1 - \theta_W)w_W$ *and* $\pi_H = \varphi_H(t^*_H)w_H - t^*_H(1 - \theta_H)w_H$*.*

This result suggests that the prices of childcare time are determined exclusively by the solutions of the cost minimization process, and not by individual preferences (except for the functions φ_I), implying that childcare activities are separated from consumption activities.[18] In this spe-

[18]This concept is known as the separation principle in agricultural economics (Benjamin, 1992).

cification, the function π_I can be viewed as a profit function, where profit is derived from the production of leisure, with a price of w_I, using childcare time as an input, with a price of $\overline{w}_I^* = (1-\theta_I)w_I$. In the empirical application, we will adopt an even more simplified formulation in which the functions $\varphi_I(t_I)$ are linear in t_I, so that the functions π_I reduce to zero.

2.2.2 Identification of the Time Cost of Children

As is usual in the literature on the identification of collective models, we first assume that individual consumptions c_W, c_H and c_K are not observed by the economist. Instead, only their sum $c_W+c_H+c_K$ is observable. The other dependent variables, l_W, t_W, l_H, t_H and m, are observed as functions of the exogenous variables. The optimal solutions of the cost minimization problem are denoted by m^*, t_W^* and t_H^* each expressed as a function of (w_W, w_H, n).

The first identification result is the following.

Proposition 3. *Assume that time feasibility conditions are not binding. The childcare cost can be recovered from the observation of m^*,t_W^*, and t_H^* as functions of (w_W, w_H, n). The technology g and the prices θ_W and θ_H can be recovered as well.*

Intuitively, the prices of pure childcare time, $w_H^* = \theta_H w_H$ and $w_W^* = \theta_W w_W$, identified under this proposition, correspond to the marginal rate of technical substitution between t_W and t_H on the one hand, and m on the other. This result is a very general consequence of the childcare technology: it holds regardless of the form of utility functions and, in particular, can be obtained without assuming A.1

2.2.3 Identification of the Monetary Cost of Children

The other structural components of the model, and in particular, the consumption of children c_K can be identified as well. This necessitates additional assumptions, though. To begin with, we note that, by application of Proposition 3, the technology g and the prices θ_I are known. Since $\theta_I = 1 - \varphi_I'(t_I)$, we can write θ_I as a function of t_I only. The functions $\varphi_I(t_I)$ can then be recovered by integrating the functions $1 - \theta_I(t_I)$, with the boundary condition $\varphi_I(0) = 0$. The full leisure time functions can then be computed as $L_I = l_I + \varphi_I(t_I)$ as well as the utility profit functions π_I.

The identification of the monetary cost of children can be achieved under various sets of assumptions. The underlying idea is always to compare the behavior of couples with children to that of couples without children, under the assumption that adult preferences remain similar regardless of whether they have children. This is a core feature — whether explicitly or implicitly — of many recent approaches, including those of Bargain and Donni (2012) or Bargain, Donni, and Hentati (2022).

To begin, we note that, in the case of childless couples, full leisure simply coincides with traditional leisure and we impose the following assumption.

Assumption A.2 The full leisure demand functions are independent of the presence of children: $L_I(w_I, \kappa_I) = L_I(w_I, \kappa_I, n)$, where $L_I = l_I$ for a couple without children.

In other words, the number of children n may affect the overall utility level of each spouse but does not alter the marginal rate of substitution between full leisure L_I and consumption c_I.

From standard results in the literature (Chiappori, 1988, 1992, or Chiappori, Fortin and Lacroix, 2002, for instance), the full leisure demand functions as well as the sharing functions κ_W and κ_H can be identified up to a unique constant from a sample of childless couples provided that certain regularity conditions are satisfied.[19] This result could serve to demonstrate that the structural model (including the full cost of children) is fully identified. However, since these aforementioned regularity conditions may be excessively demanding in practice — and given that our focus lies in estimating the cost of children, rather than in recovering the precise sharing of full income — we adopt a more tractable approach and introduce the following assumption.

Assumption A.3 The sharing functions are independent of the presence of children: $\kappa_I(w_H, w_W, Y_R) = \kappa_I(w_H, w_W, Y_R, n)$, where $Y_R = Y$ for a couple without children and $Y_R = Y + \pi_W +$

[19] The remaining constant can be determined using additional information — for example, by examining the behavior of single individuals, as shown by Lise and Seitz (2011).

$\pi_H - c_M - c_T$ for a couple with children is the 'remaining' full income, with $I = W, H$.

Taken together, Assumptions A.2 and A.3 imply that the presence of children affects full leisure only through a pure income effect — without modifying individual preferences or the intra-household allocation rule between parents. In other words, children may modify each parent's share of full income only insofar as it changes Y_R itself — that is, in exactly the same way as a change in full income would. This is the key idea behind the traditional Rothbarth approach.

Then we simply assume that the 'reduced-form' full leisure demand functions $F_I(w_H, w_W, Y_R)$, defined as

$$F_I(w_H, w_W, Y_R) = L_I(w_I, \kappa_I(w_H, w_W, Y_R)),$$

can be identified from a sample of childless couples. Due to the separable structure of the model — which stems from the collective framework — these reduced-form functions satisfy several well-known properties (Chiappori, 1988, 1992). These properties are not central for our purposes. What matters is that, under Assumptions A.2 and A.3, these reduced-form functions are invariant to the presence of children; they are identical for couples with and without children. For couples with children, the full leisure demand functions then take the form:

$$L_I = F_I(w_H, w_W, Y + \pi_W + \pi_H - c_M - c_T), \tag{4}$$

where $L_I = l_I + \varphi_I(t_I)$. If $\partial F_I / \partial Y_R \neq 0$, the reduced-form demand function can be inverted with respect to the remaining full income, which allows for exact identification of the full cost of children. Specifically, we obtain:

$$c_M = Y + \pi_W + \pi_H - c_T - F_I^{-1}\left(w_H, w_W, L_I\right), \tag{5}$$

where all the terms on the right-hand side are either observable or identifiable.

Apart from the specific way in which full leisure is defined, this approach is similar to that proposed by Bradbury (2008). Two remarks are in order. (i) Identification rests on the regularity condition $\partial F_I / \partial Y_R \neq 0$, that is, the Engel curve for full leisure (with respect to remaining full income) has a non-zero slope. (ii) As shown in expression (5), a single full leisure demand function suffices for identification; estimating both spouses' functions, however, can provide more robust estimates and a basis for specification testing.

The identification strategy relies on Assumption A.2. This assumption may be regarded as excessively strong but it can be be relaxed and replaced by the following alternative.

Assumption A.2$'$ The full leisure demand functions can be written as: $L_I = \alpha_I(w_I; n) + \beta_I(w_I, \kappa_I)$, where $L_I = l_I$ for a couple without children.

This specification implies that the number of children leaves the slope of the Engel curve unchanged but shifts its intercept — an assumption often referred to as Similarity Across Types (SAT). While still strong, it is less restrictive than Assumption A.2, which assumes identical preferences between households with and without children. The child-specific term $\alpha_I(w_I; n)$ can capture differences in preferences between parents and non-parents, as well as potential economies of scale associated with raising children. The reduced-form full leisure demand functions are then given by

$$F_I(w_H, w_W, Y_R, n) = \alpha_I(w_I; n) + \gamma_I(w_H, w_W, Y_R),$$

where $\gamma_I(w_H, w_W, Y_R) = \beta_I(w_I, \kappa_I(w_H, w_W, Y_R))$. This assumption is sufficient to identify the full cost of children provided that the regularity condition $\partial^2 F_I / \partial Y_R^2 \neq 0$ is satisfied, that is, the Engel curve for full leisure (with respect to remaining full income) has a non-constant slope. Identification proceeds along the same lines as above: the function $\gamma_I(w_H, w_W, Y_R)$ can be identified from a sample of childless couples and, once identified, can be inverted with respect to Y_R to recover the monetary cost of children.

It is important to note, however, that although Assumption A.2$'$ is less restrictive than A.2, the requirement of nonlinearity in the Engel curve is crucial for identification. This is a strong requirement and makes the strategy potentially fragile when curvature is small or difficult to estimate precisely.

3 Data and Empirical Specification

In this section, we first provide an overview of the data and then specify functional forms for the childcare technology and individual preferences.

3.1 Data

This subsection outlines the sample selection process and describes the variables used in the estimation. We also provide some preliminary, intuitive results based on reduced-form equations.

3.1.1 Sample selection

Our empirical analysis relies on the 2019 wave of the Panel Study of Income Dynamics (PSID), a longitudinal survey that provides detailed information on a broad range of topics, including income and wealth sources, wage rates, working hours, and socio-demographic characteristics. This wave also includes data on household members' time allocation — covering leisure, personal care, and childcare[20] — as well as disaggregated household expenditures, notably parents' spending on external childcare services. The 2019 wave covers 9,569 households. For our analysis, we focus on working, non-student heterosexual couples (both men and women) aged 22 to 50, with and without children. We exclude households with more than two children and those in which the youngest child is older than six years. After removing observations with incomplete data or outliers, our final sample comprises 825 households: 463 with children and 362 without.

3.1.2 Key variables

The period of analysis is one week, and all variables are calibrated accordingly. The estimation of the model relies on two key time-related variables: parental childcare time and leisure time. The PSID measures parental childcare time using the following question: "In a typical week, how many hours [do you/does [he/she]] spend caring for or looking after children?" This definition is

[20]In the PSID, time use information is collected through retrospective questions, rather than time diaries. Nevertheless, Insolera, Johnson, and Simmert (2019) find that time use measures from the PSID are generally consistent with those reported in the American Time Use Survey (ATUS).

intentionally broad, encompassing time spent caring for children even when it occurs alongside other activities — including sleep. As a result, childcare time as reported in the PSID can reach up to 168 hours per week, as shown in Table 1. These figures do not allow for an accurate assessment of the actual time cost of children, since a substantial share of this reported time may coincide with low-effort or non-exclusive tasks and therefore impose little to no opportunity cost on parents. To address this issue, we adjust the reported values downward to match more closely the American Time Use Survey (ATUS) estimates, which capture primary childcare time, defined as time spent when childcare is the respondent's main activity.[21] If this correction takes the form of subtracting a constant from all observations, it should not greatly affect the estimation of marginal prices, which depend on variation at the margin rather than on absolute levels. However, it might generate negative values for parents who report low overall childcare involvement in the PSID. To avoid this eventuality, we apply the following non-linear transformation:

$$t_I^{adj} = t_I^{psid} - k_I \times \tanh\left(\frac{t_I^{psid}}{k_I}\right)$$

where t_I^{psid} is number of hours of child care time as reported by the PSID[22] and k_I are constants chosen such that the average adjusted childcare time matches the average values reported in the ATUS.[23] This transformation has two desirable properties:[24] it preserves the zero point — i.e.,

[21]Stewart and Allard (2015) examine both primary and secondary childcare activities in the ATUS and find that, among families with children, both men and women spend nearly three times more time on secondary childcare than on primary childcare.

[22]To be precise, the PSID measure is preliminarily bounded so that the sum of weekly childcare time and working time does not exceed 168 hours — the total number of hours available in a week.

[23]We use aggregated data for couples with child under 6 from the published tables of the American Time Use Survey (ATUS) for the period 2015–2019 (Table 9 in Bureau of Labor Statistics, News Release, 10:00 a.m. (EDT) Thursday, June 25, 2020). In these aggregated data, childcare time refers exclusively to primary activities — that is, activities identified by respondents as their main activity at the time. It includes physical care, education-related activities, reading to or with children, playing or engaging in hobbies with children, and travel related to caring for and helping household children.

[24]Recall that $\tanh(x) = (e^x - e^{-x}) / (e^x + e^{-x})$.

Table 1: Parents' Weekly Childcare Time (in Hours, with and without Adjustment) and Weekly Childcare Expenditure (in Dollars)

	Mean	StDev	Min	50%	Max	Pzeros
Father's Original PSID Childcare Time	35.12	39.72	0.00	20.00	168.00	0.02
Mother's Original PSID Childcare Time	53.65	46.39	0.00	40.00	168.00	0.01
Father's Adjusted Childcare Time	9.94	21.05	0.00	0.97	116.71	0.02
Mother's Adjusted Childcare Time	19.18	28.65	0.00	6.78	117.56	0.01
Childcare Expenditure	99.91	127.46	0.00	49.96	769.23	0.35

Notes: Mean = average value, StDev = standard-deviation, Min = minimum value, 50% = median, Max = maximum value, Pzeros = proportion of zeros.

$t_I^{adj} = t_I^{psid}$ when $t_I^{psid} = 0$ — and it approximates a constant downward shift for large values of reported time, since $t_I^{adj} \approx t_I^{psid} - k_I$ when t_I^{psid} is large. As shown in Table 1, the proportion of zero values is negligible and remains unchanged under both definitions of childcare time. However, under the adjusted definition, the average number of hours is substantially lower — about 10 hours per week for fathers and 19 hours for mothers.

The second key time-related variable is leisure time. The PSID includes a direct question on usual weekly leisure hours: "In a typical week, how many hours [do you/does [he/she]] spend doing leisure activities for enjoyment, for example, watching TV, doing physical activities that [you enjoy / he or she enjoys], going online, or spending time with friends?" This measure of leisure time is problematic because it does not match the definition used in the theoretical model. It is relatively narrow and excludes activities not typically labeled as leisure — such as cooking, shopping, or even sleeping — even though these may yield utility comparable to leisure. For these reasons, we follow the standard practice in the literature and define leisure time as non-market time. Specifically, weekly leisure time is computed as $l_I = T - h_I - t_I$, where T is fixed at 119.7 hours[25] per week, and h_I denotes hours spent in paid work. Once the $\varphi_I(t_I)$ will be estimated, it will be possible to compute the full leisure time as $L_I = l_I + \varphi_I(t_I) = T - h_I - (1 - \varphi_I(t_I))t_I$, where $\varphi_I(t_I)$ corresponds to the part of parental childcare time perceived as leisure.

[25]This figure corresponds to $24 - 6.9 = 17.1$ hours per day, where 6.9 hours is the average sleeping time.

Table 2: Descriptive Statistics

	Without Child		With Children	
	Mean	StDev	Mean	StDev
Father's Adjusted Childcare Time	0.00	0.00	9.94	21.05
Mother's Adjusted Childcare Time	0.00	0.00	19.18	28.65
Childcare Expenditure	0.00	0.00	99.91	127.46
Number of Children	0.00	0.00	1.56	0.50
Dummy for Child under 3	0.00	0.00	0.74	0.44
Husband's Leisure Time	77.03	12.28	66.19	22.65
Wife's Leisure Time	82.49	12.30	66.98	28.80
Husband's Hourly Wage	30.71	44.51	31.97	25.89
Wife's Hourly Wage	24.36	16.36	29.48	26.25
Full Expenditure (in thousand dollars)	5.52	4.15	6.31	4.25
Husband's Education in Years	14.24	2.26	14.38	2.39
Wife's Education in Years	14.84	2.10	15.15	1.87
Husband's Age in Years	36.71	7.82	34.21	5.33
Wife's Age in Years	35.58	7.84	32.88	5.04

Notes: StDev = standard-deviation.

Finally, the PSID elicits childcare expenditures using the question: "How much did you (and your family living there) pay for child care in 2018?" To obtain weekly childcare expenditures, we divide the reported annual amount by 52. As shown in Table 1, the proportion of zeros is substantial but but does not compromise the feasibility of the estimation.

Summary statistics for the variables used in the estimations are presented in Table 2, separately for couples without children and couples with children. On average, childless women report about 15 more hours of leisure per week than mothers, while childless men report around 11 more hours than fathers. For couples with children, average weekly childcare expenditures amount to approximately $100, with a large standard deviation reflecting substantial heterogeneity in spending. Aside from differences directly related to the presence of children, the two groups of couples exhibit broadly similar average characteristics across the remaining variables. These include spouses' hourly wages (directly provided by the PSID), education levels (measured as the highest grade of school completed), ages (in years). The variables also include the household full expenditure (defined as the sum of household total expenditure on goods and services and expenditure in leisure time computed with spouses' wage rates as prices) that is used in the estimation instead of the full income to ensure model consistency with a life-cycle framework (Blundell and MaCurdy, 1999).

3.1.3 Preliminary regressions

Before turning to the structural model, we begin by presenting reduced-form OLS estimates of the childcare time and expenditure equations. The estimates are reported in Table 3. The effects of parental hourly wages on childcare expenditures are positive and decreasing, while the effects on childcare time are negative and increasing (that is, the curves are concave with respect to wages). These estimates allow for a back-of-the-envelope approximation of the price of parental time devoted to childcare. Evaluated at the average point of the sample (that is, $w_H = 31.97$ and $w_W = 29.48$ using figures in Table 2), a \$10 increase in the father's (mother's) hourly wage is associated with an increase of approximately \$6.44 (\$15.60) in spending on childcare services. At the same time, a \$10 increase in the father's (mother's) hourly wage is associated with a reduction of approximately 0.34 (2.05) hours in the father's (mother's) time devoted to childcare. This implies that the father substitutes one hour of childcare time with \$18.94($\approx 6.44/0.34$) of childcare services — interpreted as the value of the father's childcare time. Relative to the value of his wage, this gives $\theta_H = 0.59 \approx 18.94/31.97$. Similarly, the variation in the value of the mother's childcare time is \$7.61($\approx 15.60/2.05$), giving $\theta_W = 0.26 \approx 7.61/29.48$.

3.2 Empirical Specification

In this subsection, we present the functional form for the childcare technology and the demands for full leisure. We also incorporate observable and unobservable heterogeneity.

3.2.1 The Childcare Technology

The concept of childcare technology captures the need for coordination between parents in providing care for their children. Instead of specifying the technology, we directly model the

Table 3: OLS estimations of reduced-form childcare time and expenditure equations

	Expenditure		Father's Time		Mother's Time	
	Est	StErr	Est	StErr	Est.	StErr
Constant	31.52	16.72	13.18	3.00	26.57	3.80
Father's Wage	0.74	0.76	-0.04	0.09	-0.05	0.13
Squared Father's Wage / 100	-0.15	0.70	0.01	0.04	0.03	0.05
Mother's Wage	1.92	0.55	-0.10	0.07	-0.27	0.11
Squared Mother's Wage / 100	-0.61	0.33	0.05	0.03	0.11	0.04
Father's Age	3.31	8.41	0.27	1.83	-0.66	2.28
Mother's Age	11.83	9.63	-1.60	1.96	-1.51	2.48
Father's Education	11.08	5.70	-0.78	1.20	-2.64	1.51
Mother's Education	10.18	5.67	-0.15	1.07	0.50	1.74
Number of Children	19.47	5.52	-0.56	1.01	-1.20	1.32
Dummy for Child under 3	6.96	5.11	0.24	1.06	0.33	1.40

Notes: All demographic variables are standardized. Est = estimate, StErr = standard-error.

time cost function. We begin by considering the case of interior solutions.

If $m > 0$ (i.e., the solution is interior), the time cost function is as follows:

$$e_T = \alpha^* + \beta_W^* w_W^* + \beta_H^* w_H^* + 2\gamma_W \sqrt{w_W^*} + 2\gamma_H \sqrt{w_H^*} + 2\gamma_{WH} \sqrt{w_W^* w_H^*}, \tag{6}$$

where α^*, β_W^*, β_H^*, γ_W, γ_H and γ_{WH} are parameters, w_H^* and w_W^* are the price of parents' pure childcare time as previously explained, which is supposed to be proportionate to hourly wage rates: $w_I^* = \theta_I w_I$ for some positive parameter θ_I. This specification, which is inspired from the Generalized Leontieff of Diewert (1971), is globally consistent with the theory (increasing and concave with respect to prices) whenever $\beta_W^* \geq 0$, $\beta_H^* \geq 0$, $\gamma_H \geq 0$, $\gamma_W \geq 0$ and $\gamma_{WH} \geq 0$. Its appeal is that it accommodates both substitution and complementarity between parents' inputs through the interaction term $\sqrt{w_W^* w_H^*}$, albeit potentially at the cost of the global consistency.

From Shephard's Lemma, the solution of the cost minimization problem is:

$$t_H = \beta_H^* + \gamma_H \frac{1}{\sqrt{w_H^*}} + \gamma_{WH}\sqrt{\frac{w_W^*}{w_H^*}} \tag{7}$$

$$t_W = \beta_W^* + \gamma_W \frac{1}{\sqrt{w_W^*}} + \gamma_{WH}\sqrt{\frac{w_H^*}{w_W^*}} \tag{8}$$

if $m > 0$. Defining $\beta_I^* = \beta_I \boldsymbol{z}_{T,I} + v_I$, where $\boldsymbol{z}_{T,I}$ are control variables, v_I is an error term, and using $w_I^* = \theta_I w_I$, we obtain the childcare time equations:

$$t_H = \beta_H \boldsymbol{z}_{T,H} + \gamma_H \frac{1}{\sqrt{\theta_H w_H}} + \gamma_{WH}\sqrt{\frac{\theta_W w_W}{\theta_H w_H}} + v_H \tag{9}$$

$$t_W = \beta_W \boldsymbol{z}_{T,W} + \gamma_W \frac{1}{\sqrt{\theta_W w_W}} + \gamma_{WH}\sqrt{\frac{\theta_H w_H}{\theta_W w_W}} + v_H. \tag{10}$$

To incorporate heterogeneity in the demand for external childcare services, we similarly write: $\alpha^* = \alpha \boldsymbol{z}_T + u$ where $\boldsymbol{z}_T$ are control variables and u is an error term. Using $m = e_T - w_H t_H - w_W t_W$ and expressions (6), (9) and (10), we obtain the childcare expenditure equation:

$$m = \begin{cases} m^* & \text{if } m^* \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

where

$$m^* = \alpha \boldsymbol{z}_T + \gamma_W \sqrt{\theta_W w_W} + \gamma_H \sqrt{\theta_H w_H} + u \tag{12}$$

If $m = 0$ (i.e., the solution is at a corner), parents' childcare time equations switch regime and the prices of parents' pure childcare time must satisfy the following condition:

$$\alpha \boldsymbol{z}_T + \gamma_W \sqrt{w_W^*} + \gamma_H \sqrt{w_H^*} + u = 0, \tag{13}$$

with $w_H^* = \nu \theta_H w_H$ and $w_W^* = \nu \theta_W w_W$ for some function ν, to guarantee that the childcare constraint is satisfied.[26] This function can be recovered from the inversion of (13).

Finally, the error terms (u, v_W, v_H) are assumed to be jointly normally distributed with mean zero and an unconstrained variance-covariance matrix. In our estimations, the control variables $\boldsymbol{z}_{T,H}$, $\boldsymbol{z}_{T,W}$ and $\boldsymbol{z}_T$ include a constant, the spouses' age, the spouses' education, the number of children, as well as a dummy for the presence of children under the age of 3.

3.2.2 The Full Leisure Demand Equations

To begin with, we note that $\varphi_I(t_I) = (1 - \theta_I)t_I$ in spouses' utility functions since θ_I is constant, as previously mentioned. As a result, the total leisure enjoyed by each spouse is defined as $L_I = l_I + (1 - \theta_I)t_I$. With a linear specification, the prices of pure and leisure-like childcare time have a simple interpretation: the term $\theta_I t_I$ can be seen as the fraction or percentage of childcare time that is perceived as pure labor, while the term $(1 - \theta_I)t_I$ as the fraction that is perceived as pure leisure. The linear specification also guarantees that the utility profit functions π_W and π_H are always equal to zero and can therefore be omitted from the analysis.

As previously discussed, our empirical model employs full expenditure — denoted as Y — rather than full income as the primary explanatory variable. The time cost of children is derived from the childcare technology and defined by expression (6), while the monetary cost is specified as:

$$c_M = \log\left(1 + \exp\left(\kappa_0 + \kappa_H w_H + \kappa_W w_W + \kappa_Y w_Y\right)\right) \times \sqrt{n} \tag{14}$$

where κ_0, κ_H, κ_W, and κ_Y are parameters, and n is the number of children. The transformation in (14) is commonly referred to as the softplus function in machine learning. This function

[26] See the proofs of Proposition 2 and 3 for the derivation of w_H^* and w_W^*.

ensures that monetary costs remain strictly positive, while offering a smooth and differentiable alternative to the simple maximum operator or the exponential function.

The remaining full expenditure Y_R is equal to Y for couples without children and to $Y - c_M - c_T$ for couples with children. For couples with and without children, the full leisure demand equations are specified either as:

(i) a quadratic function of remaining full expenditure:

$$L_H = a_H \boldsymbol{z}_{L,H} + b_H w_H + c_H w_W + d_H Y_R + e_H Y_R^2 + \varepsilon_H \tag{15}$$

$$L_W = a_W \boldsymbol{z}_{L,W} + b_W w_W + c_W w_H + d_W Y_R + e_W Y_R^2 + \varepsilon_W \tag{16}$$

where a_I, b_I, c_I, d_I and e_I are parameters while $\boldsymbol{z}_{L,I}$ are control variables, and ε_I is an error term, or as:

(ii) a quadratic function of remaining full expenditure with a demographic shift:

$$L_H = a_H(n) \boldsymbol{z}_{L,H} + b_H(n) w_H + c_H(n) w_W + d_H Y_R + e_H Y_R^2 + \varepsilon_H \tag{17}$$

$$L_W = a_W(n) \boldsymbol{z}_{L,W} + b_W(n) w_F + c_W(n) w_H + d_W Y_R + e_W Y_R^2 + \varepsilon_W \tag{18}$$

where $a_I(n) = a_I + a'_I \delta(n)$, $b_I(n) = b_I + b'_I \delta(n)$ and $c_I(n) = c_I + c'_I \delta(n)$ are functions of n, a'_I, b'_I and c'_I are parameters and $\delta(n)$ is dummy variable equal to one if $n > 0$ and 0 otherwise. This specification allows the presence of children to shift the full leisure demand function, while leaving the slope of the Engel curves unchanged.

In our estimations, the control variables $\boldsymbol{z}_{L,H}$ and $\boldsymbol{z}_{L,W}$ include a constant, spouses' education and spouses' age. The error terms $(\varepsilon_W, \varepsilon_H)$ are assumed to have mean zero and an unconstrained variance-covariance matrix. Finally, it is worth noting that all the three systems of equations are consistent with the collective framework, i.e., the aforementioned properties of collective leisure demand functions (Chiappori, 1988, 1992) are automatically satisfied.

4 Estimation Method and Empirical Results

4.1 Estimation Method

In this subsection, we first present the two stage estimation method based on the Maximum Likelihood method and the Three-Stage Least Square method.

While the five equations could be estimated simultaneously, we simplify the estimation procedure by taking advantage of the two-stage decision-making structure and estimate the system recursively.[27] The two stages rely on different samples: the first stage is estimated using only couples with children, while the second stage is estimated using the full sample of couples. In the first stage, the three-equation system for the childcare technology, represented by (9)–(12), is estimated. In the data, a substantial proportion of parents report zero expenditure on paid childcare services, whereas the number of parents reporting zero hours of childcare time per week is negligible.[28] We therefore assume that the two equations for parental childcare time are uncensored, whereas the equation for childcare expenditure is censored at zero.[29] The resulting system is thus a mixed specification, combining linear and Tobit components, and is estimated by the Maximum Likelihood (ML) method. In the second stage, the time cost and the value of both full leisure times are computed for all observations using $\widehat{\theta}_W$ and $\widehat{\theta}_H$ obtained in the first stage. The two-equation system for spouses' full leisure demand, represented by (15)–(16) or (17)–(18), is then estimated by the Three-Stage Least Squares (3SLS) method. This estimation method accounts for the potential endogeneity of the full expenditure and the time cost

[27] A further benefit is that potential specification errors are not conflated across stages, allowing for a clean identification and estimation of childcare-time prices.

[28] The few observed zeros in childcare time may reflect measurement errors.

[29] However, we refrain from modeling regime switching in the childcare time equations so as to avoid a major source of nonlinearity.

computed in the first stage.[30] To do so, we use the fitted value of time cost obtained from the first stage estimation and the fitted value of the full expenditure obtained by OLS using the following instruments: a constant, spouses' education, spouses' age, spouses' hourly wages, weekly nonlabor income and its square, separately for couples with and without children.

4.2 Empirical Results

4.2.1 Estimates of the Childcare Equations

In Table 4, the estimated parameters for the childcare time and expenditure equations are reported for three different samples or specifications. The estimates and standard errors in columns A are based on the sample described in Table 2 and those in columns B are based on a restricted sample constructed by dropping the 20 largest and 20 smallest observations for full expenditure and for spouses' hourly wages. This restriction aims at mitigating the potential influence of outliers, ensuring that estimates reflect general patterns rather than a few extreme cases. The estimates and standard errors in columns C are based on the same sample as for columns A except that it uses the original PSID childcare time values rather than the adjusted ones. The results show that, overall, the different samples provide very similar parameter estimates (the intercepts in columns C are much larger because they absorb the adjustment constants k_H and k_W) but the associated standard errors in columns B are noticeably larger. Control variables are generally not significant with a few exceptions. Childcare expenditure rises significantly with parental education and with the number of children, whereas childcare time shows no comparable pattern. As expected, spouses' hourly wages are negatively associated with the time spent on childcare and positively associated with expenditure on paid childcare services. More precisely, the derivative of the father's childcare time equation with

[30] There are several reasons to believe that full expenditure and time cost may be endogenous. Most notably, measurement errors in working time and childcare time — and, by construction, in leisure time — are directly reflected in the computed values of full expenditure and childcare cost, respectively.

Table 4: ML Estimates of Childcare Time and Expenditure Equations

		A		B		C	
Param.	Variable	Est	StErr	Est	StErr.	Est	StErr.
α_0	Intercept	-99.28	28.71	-83.67	39.24	-98.38	28.71
α_1	Husband's Education	15.81	8.99	10.36	7.82	15.59	9.01
α_2	Wife's Education	15.16	9.41	5.63	8.19	15.34	9.41
α_3	Husband's Age	6.99	13.19	13.67	11.23	6.99	13.19
α_4	Wife's Age	19.56	13.77	8.52	11.72	19.31	13.76
α_5	Dummy for Child under 3	9.40	8.08	-7.14	7.07	9.42	8.07
α_6	Number of Children	30.08	7.87	17.25	6.92	30.05	7.86
$\beta_{H,0}$	Intercept	6.28	2.40	7.13	3.57	27.40	4.62
$\beta_{H,1}$	Husband's Education	-0.54	1.15	0.11	1.21	-0.70	2.18
$\beta_{H,2}$	Wife's Education	0.01	1.20	-0.52	1.28	0.11	2.26
$\beta_{H,3}$	Husband's Age	0.25	1.67	-0.15	1.74	0.07	3.15
$\beta_{H,4}$	Wife's Age	-1.35	1.74	-0.94	1.81	-2.79	3.29
$\beta_{H,5}$	Dummy for Child under 3	0.32	1.02	0.27	1.11	1.63	1.93
$\beta_{H,6}$	Number of Children	-0.56	0.99	-1.00	1.06	-0.13	1.87
$\beta_{W,0}$	Intercept	13.12	2.54	13.87	6.17	44.72	4.10
$\beta_{W,1}$	Husband's Education	-2.55	1.55	-2.02	1.65	-3.96	2.51
$\beta_{W,2}$	Wife's Education	-0.04	1.57	-0.64	1.75	0.72	2.56
$\beta_{W,3}$	Husband's Education	-0.34	2.26	1.74	2.38	-0.88	3.67
$\beta_{W,4}$	Wife's Education	-1.74	2.33	-3.33	2.47	-2.92	3.80
$\beta_{W,5}$	Dummy for Child under 3	0.40	1.39	0.20	1.51	0.67	2.26
$\beta_{W,6}$	Number of Children	-1.17	1.34	-1.23	1.45	-1.90	2.17
γ_H		20.67	6.74	16.95	9.71	27.50	9.85
γ_W		27.40	6.40	22.53	16.55	31.45	8.59
$\gamma_{W,H}$		-3.28	2.49	-2.66	3.28	-4.07	4.61
θ_H		0.35	0.21	0.42	0.55	0.20	0.12
θ_W		0.44	0.26	0.66	0.84	0.32	0.24

Notes: All demographic variables are standardized. A = the benchmark sample, B = the sample without extreme values, C = the sample with original PSID childcare time measures.

respect to his hourly wage is given by:

$$\frac{\partial t_H}{\partial w_H^*} = -\frac{1}{2}\gamma_H \left(w_H^*\right)^{-3/2} - \frac{1}{2}\gamma_{HW} \left(w_W^*\right)^{1/2} \left(w_H^*\right)^{-3/2}$$

which corresponds to a numerical effect of approximately -0.12. In other words, a one-dollar increase in the father's hourly wage is associated with about 0.12 fewer hours of childcare per week ($\approx$ 7 minutes), holding the mother's hourly wage and other covariates fixed. This effect is moderate but strongly nonlinear, so that at the first decile of the wage distributions the marginal effect is roughly -0.91 hours ($\approx$ 55 minutes). Similarly, the derivative with respect to the mother's hourly wage is given by:

$$\frac{\partial t_H}{\partial w_W^*} = -\frac{1}{2}\gamma_{HW} \left(w_H^*\right)^{-1/2} \left(w_W^*\right)^{-1/2}$$

Computed at the average point of the sample, the numerical effect is equal to -0.14 ($\approx$ 8 minutes), holding the father's hourly wage and other covariates fixed. The negative cross-wage effect implies that the two parental time inputs can be seen as Hicks–Allen complements. Finally, the derivative of the mother's childcare time equation with respect to her hourly wage is equal to -0.18 hours ($\approx$ 11 minutes) while the derivative with respect to the father's hourly wage is, by symmetry, also equal to -0.14 ($\approx$ 8 minutes).[31] The fact that the parameters γ_{WH} is negative implies that the cost function is not globally consistent. Unreported results indicate that the error terms in the parents' childcare time equations are positively correlated. The mother'schildcare time error term is also positively correlated with that of childcare expenditures, while the corresponding correlation for the father is insignificant.

[31] The number of studies that explicitly examine the relationship between childcare time and hourly wages is very limited. They generally do not find significant effects of hourly wages or confirm the negative effect we obtain (Bloemen and Stancanelli, 2014, for France; Kalenkoski, Ribar, Stratton, 2009, for U.K.; Hallberg and Klevmarken, 2003, and Gustafsson and Kjullin, 1994, for Sweden). Among the very few available studies based on U.S. data, Kimmel and Connelly (2007) present the surprising finding, with the American Time Use Survey (ATUS), that an increase in wage rates is associated with an increase in time spent on childcare. They interpret this as an income-effect mechanism: as wages rise, households demand higher-quality childcare activities. Note that their sample includes both couples and single-parent households, with multiple and older children.

The most important parameters in the model are the price-scales coefficient θ_H and θ_W. Under the linear specification of process benefits, these parameters can be interpreted as the proportion of childcare time that is perceived as pure labor by the mother and the father, respectively. Using the benchmark sample, our results suggest that about 44% of the time mothers spend caring for their children is treated as labor, while the remainder resembles leisure, while this proportion is lower for fathers and equal to about 35%. Using the sample which trims extreme values (columns B), the coefficients are somewhat higher but not significantly different and, using the original PSID childcare time measures (columns C), they are slightly smaller. Alternatively, these parameters can also be interpreted as indicators of parental productivity in domestic tasks. Holding wage rates constant, a higher θ implies greater productivity in equilibrium. For instance, a mother with $\theta_W = 0.44$ is more productive in domestic childcare tasks than a father with $\theta_H = 0.35$.

To assess robustness, we compare the estimated price-scale coefficients with the back-of-the-envelope approximations obtained from preliminary OLS specifications and find broadly similar magnitudes. We also re-estimate models with a more parsimonious set of covariates; the results change little. In addition, we fit a more flexible cost function that includes a quadratic in hourly wages; the added terms are statistically indistinguishable from zero at the 5% level. That said, the estimated price-scales coefficients might be subject to downward bias for at least two reasons. First, the demand for formal childcare services may be rationed — either because the couple lives in a region with limited availability of childcare facilities, or because available services do not match the household's preferences in terms of quality, schedule, or accessibility. Second, the actual cost of using formal childcare services may be higher than what is reported in the PSID, as it may involve fixed, unobserved costs — such as commuting to the care provider, or the time and effort required to coordinate logistics, including scheduling and drop-off/pick-up arrangements. These factors could attenuate the sensitivity of the demand for childcare services and lead to an underestimation of price-scales coefficients.

Despite these potential limitations, the estimated price-scales coefficients appear plausible and provide a solid basis for computing both spouses' full leisure time and the time cost of children.

4.2.2 Estimates of the Full Leisure Demand Equations

This subsection sequentially presents the parameters estimates of Engel curves and the monetary cost of children. In Table 5, the columns A display the estimates and standard errors obtained from the subsample of childless couples — and thus does not involve the estimation of the monetary cost of children. The columns B and C correspond, respectively, to the estimates and standard errors of the standard Quadratic Engel curve specification and of the Quadratic, Demographic-Shift Engel curve specification. In the bottom panel of the table are displayed the estimated parameters of the monetary cost functions.

Using the childless sample, we find a positive but weakly significant relationship between full expenditure and full leisure, defined as non-working time. A Wald test does not reject the null hypothesis of a flat Engel-curve slope in either equation. Including couples with children leaves the estimates essentially unchanged but reduces the standard errors considerably, owing to the larger sample, greater variation in both full leisure and full expenditure, and the strong instrument generated by the fitted time cost. Intuitively, the large differences in full leisure between childless couples and parents is attributed in the model to the full cost of children, a fraction of which is represented by the time cost. The parameters are generally significantly different from zero, with the notable exception of the quadratic terms. In particular, a $1,000 increase in full expenditure is associated (ignoring quadratic terms to simplify) with an additional 3.4 hours of full leisure for men and and 4.6 hours for women, while higher hourly wages are associated with less full leisure for both spouses. By contrast, the parameters of the monetary cost function are imprecisely estimated (and adding covariates substantially increases standard errors). As shown below, average costs can be recovered with reasonable precision, but the covariate effects on those costs are weakly identified.

The columns C of Table 5 report estimates and standard errors for the Quadratic, Demographic-Shift Engel curve specification. Overall, the findings are broadly consistent with the preceding ones, albeit with slightly larger standard errors. These estimates should, however, be interpreted with caution. The parameters that shift Engel curves are effectively not statistically different from zero — either individually or jointly, according to a Wald test. In addition, identification

Table 5: 3SLS Estimates of Full Leisure Demand Equations

		A		B		C	
Param.	Variables	Est.	St.Err.	Est.	St.Err.	Est.	St.Err.
$a_{H,0}$	Intercept	74.47	12.32	71.79	2.73	71.80	2.81
$a_{H,1}$	Husband's Education	-1.04	2.61	-1.51	0.56	-1.14	0.87
$a_{H,2}$	Wife's Education	0.63	2.37	0.34	0.57	0.52	0.78
$a_{H,3}$	Husband's Age	-0.56	3.60	-0.75	0.89	-0.65	1.20
$a_{H,4}$	Wife's Age	0.05	3.52	0.22	0.91	-0.05	1.17
b_H	Husband's Hourly Wage	-0.01	0.82	-0.31	0.15	-0.14	0.18
c_H	Wife's Hourly Wage	-0.14	0.74	-0.32	0.14	-0.26	0.16
d_H	Full Expenditure	1.03	8.09	3.41	1.46	2.68	1.63
e_H	Squared Full Expenditure	-0.00	0.13	0.00	0.03	-0.02	0.03
$a'_{H,0}$	Intercept	.	.	.	.	-2.09	3.03
$a'_{H,1}$	Husband's Education	.	.	.	.	-0.58	1.14
$a'_{H,2}$	Wife's Education	.	.	.	.	-0.26	1.15
$a'_{H,3}$	Husband's Age	.	.	.	.	-0.14	1.79
$a'_{H,4}$	Wife's Age	.	.	.	.	0.83	1.85
b'_H	Husband's Hourly Wage	.	.	.	.	-0.10	0.06
c'_H	Wife's Hourly Wage	.	.	.	.	0.11	0.06
$a_{W,0}$	Intercept	80.87	13.21	79.15	3.40	80.01	3.56
$a_{W,1}$	Husband's Education	-0.64	2.80	-0.06	0.62	-0.72	0.95
$a_{W,2}$	Wife's Education	-0.72	2.55	-1.48	0.64	-0.71	0.86
$a_{W,3}$	Husband's Age	-0.55	3.87	-1.50	0.97	-0.51	1.31
$a_{W,4}$	Wife's Age	0.28	3.78	0.75	1.01	0.26	1.28
b_W	Husband's Hourly Wage	-0.45	0.88	-0.48	0.19	-0.49	0.21
c_W	Wife's Hourly Wage	-0.55	0.80	-0.52	0.17	-0.38	0.23
d_W	Full Expenditure	4.20	8.68	4.57	1.92	3.96	2.25
e_W	Squared Full Expenditure	0.03	0.14	0.02	0.04	0.01	0.04
$a'_{W,0}$	Intercept	.	.	.	.	-3.61	3.89
$a'_{W,1}$	Husband's Education	.	.	.	.	1.25	1.26
$a'_{W,2}$	Wife's Education	.	.	.	.	-1.22	1.31
$a'_{W,3}$	Husband's Age	.	.	.	.	-2.06	1.98
$a'_{W,4}$	Wife's Age	.	.	.	.	0.96	2.07
b'_W	Husband's Hourly Wage	.	.	.	.	0.09	0.08
c'_W	Wife's Hourly Wage	.	.	.	.	0.02	0.07
κ_0		.	.	3.17	2.18	3.31	3.32
κ_H		.	.	-0.17	0.13	-0.20	0.21
κ_W		.	.	-0.09	0.13	-0.04	0.20
κ_Y		.	.	0.47	1.26	0.33	1.96

Notes: A = the childless sample, B = the Quadratic Engel curve specification, C = the Quadratic, Demographic-Shift Engel curve specification.

hinges on the nonlinearity of the monetary cost function because the quadratic terms in the Engel curves are not statistically different from zero. When that function is replaced by a simple linear form, the standard errors increase dramatically. With this caveat in mind, however, we note that both intercept shifts are negative, suggesting that the child effect is partially absorbed by these shifts rather than by the monetary cost. Since the model is over-identified and the parameters of the monetary cost equations can be identified from a single full leisure equation, we also estimated a version of the model allowing these parameters to differ by gender. Based on a distance test using the standard Quadratic Engel curve specification, the resulting chi-squared statistics is equal to 6.096 with four degrees of freedom, suggesting that allowing for gender-specific monetary cost parameters does not significantly improve the fit.

4.2.3 Estimates of the time and monetary cost of children

The characteristics of the distribution of time and monetary costs, across the different specifications and for one and two children compositions, are reported in Tables 6. At the sample mean, the time cost for a single child is sizable — about \$ 430 per week (row A), \$ 520 when trimming extreme values (row B), and \$ 810 when using the original PSID childcare time measure (row C) — representing between 23% and 35% of total spousal labor income. The differences between the first two specifications are mainly driven by variations in the price of childcare time, evaluated by the estimation of childcare equations. The third specification relies on a time measure that includes passive childcare (e.g., supervisory time) which likely leads to an overstatement of the true time cost. In all cases, the time cost decreases sligthly with the number of children, both because childcare time is fairly insensitive to family size and because the one-child group includes high hourly-wage outliers that inflate the imputed cost. The density function of time costs under the benchmark specification, pooling the sample of couples with one and two children, is shown in Figure 7, where a distinct right-skew (long right tail) is observed.

Finally, the monetary cost for a single child, evaluated at its sample mean, is estimated at around \$740 per week for the standard Quadratic Engel curve specification (row A) and \$720 per week for the Quadratic, Demographic-Shift Engel curve specification (row B) but it increases to more than \$1000 for two children. The density of monetary costs under the Quadratic Engel curve

Table 6: Descriptive Statistics for the Distribution of the Weekly Time and Monetary Cost of Children (in Thousands of Dollars per Week)

	Children	count	mean	std	min	25%	50%	75%	max
		Weekly time cost in thousands dollars							
A	1	205	0.43	0.64	0.00	0.08	0.23	0.53	5.18
	2	258	0.40	0.50	0.00	0.10	0.25	0.49	3.62
B	1	161	0.52	0.71	0.00	0.12	0.27	0.67	5.04
	2	201	0.41	0.48	0.00	0.11	0.27	0.50	3.37
C	1	205	0.81	0.90	0.00	0.33	0.57	0.97	8.07
	2	258	0.76	0.73	0.00	0.32	0.56	0.94	6.15
		Weekly monetary cost in thousands dollars							
A	1	203	0.74	0.77	0.00	0.09	0.46	1.20	2.86
	2	252	1.08	1.07	0.00	0.19	0.75	1.69	4.39
B	1	203	0.72	0.81	0.00	0.05	0.38	1.14	3.49
	2	252	1.01	1.10	0.00	0.10	0.62	1.55	4.66

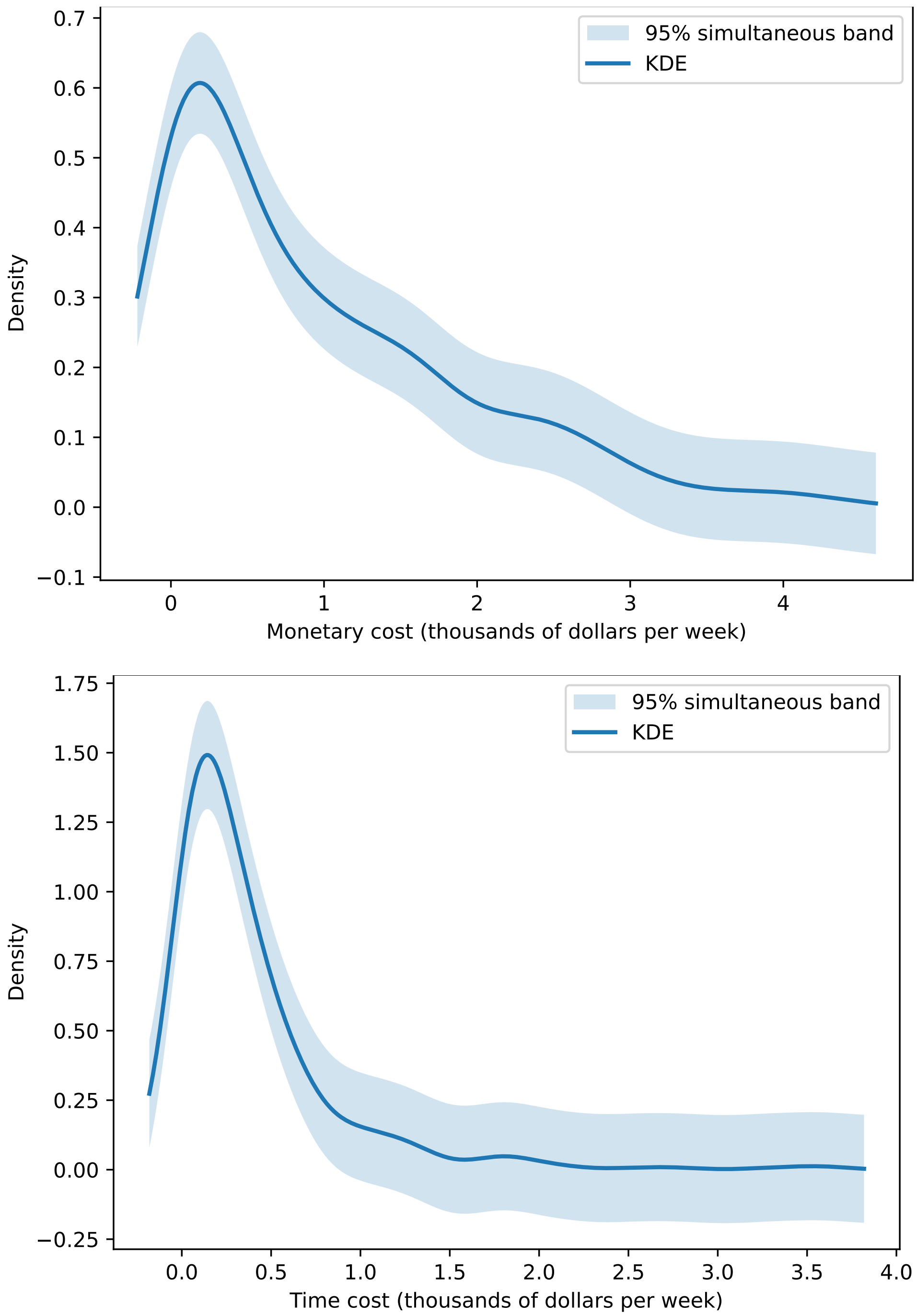
Notes: For time cost, A = the benchmark sample, B = the sample without extreme values, C = the sample with original PSID childcare time measures. For monetary cost, B = the Quadratic Engel curve specification, C = the Quadratic, Demographic-Shift Engel curve specification.

specification is shown in Figure 7 and is likewise distinctly right-skewed.

5 Conclusion

In this paper, we have developed a static model to evaluate the full cost of children, including both a monetary component and a time component. A crucial point of our study lies in recognizing that the price of time used to assess the time cost of children is not necessarily equivalent to the parents' wage rate. Instead, it depends on how parents perceive their time spent with their children — whether they consider it closer to leisure or to labor. To the best of our knowledge, this is the first study to explicitly make this distinction and to provide estimates of the full cost of children that account for this nuanced valuation of time.

Table 7: Distribution of Time and Monetary Costs (in thousands of dollars per week)



Estimating the model requires detailed information on both the time parents devote to their children and their spending on external childcare. We implement it using the 2019 PSID, which provides the necessary data. Recovering the full cost of children raises two challenges, though. First, the full leisure demand equations are only weakly responsive to changes in full expenditure — the associated Engel curves are relatively flat — so identification based on leisure alone is potentially weak. On this point, it is important to note that recovering the full cost does not strictly require leisure time. Alternative goods (such as clothing, which typically displays a steeper Engel curve) could serve the same identification purpose. Implementing this approach, however, would require endogenizing market hours and specifying clothing expenditure as a function of full expenditure and hourly wages, thereby departing from standard empirical practice and motivating a distinct research agenda[32] Second, full expenditure must be instrumented, and credible instruments are scarce. Our theoretical framework nonetheless delivers a strong instrument — the fitted value of the child time cost obtained from the childcare equations — though it is model-dependent.

Notwithstanding these caveats, our baseline results indicate that one hour of the mother's childcare time is priced at approximately 44% of her hourly wage, and one hour of the father's at 35% of his hourly wage. The prices of childcare time differ from current hourly wages because a fraction of childcare time can be assimilated to pure leisure. The resulting time cost of children, based on these measures of childcare time, is substantial — approximately $430 per week for a single child. The corresponding monetary cost averages about $720 per week, implying a full cost exceeding $1,000 per week, substantially higher than typical estimates in the literature. In addition, these estimates do not capture longer-term costs, such as career interruptions or reduced human capital accumulation among parents, particularly mothers. The total cost of children may therefore be considerably higher than the short-run estimates reported here. While these long-term considerations are beyond the scope of this study, they represent a promising avenue for future research.

[32]As previously, said, our procedure first recovers the time cost and then the full cost of children; the monetary cost is obtained residually as the difference between the two. Alternatively, however, one could recover the monetary cost in the second step and back out the full cost as the sum of the time and monetary components.

Appendix : Proofs

Proof of Proposition 1.

The first order conditions of the optimization problem $\bar{\mathrm{P}}$ are:

$$l_W : \phi_W \frac{\partial u_W}{\partial l_W} - \lambda_1 w_W = 0 \tag{A.1}$$

$$t_W : \phi_W \frac{\partial u_W}{\partial t_W} - \lambda_1 w_W - \lambda_2 \frac{\partial g}{\partial t_W} = 0 \tag{A.2}$$

$$c_W : \phi_W \frac{\partial u_W}{\partial c_W} - \lambda_1 = 0 \tag{A.3}$$

$$l_H : \phi_H \frac{\partial u_H}{\partial l_H} - \lambda_1 w_H = 0 \tag{A.4}$$

$$t_H : \phi_H \frac{\partial u_H}{\partial t_H} - \lambda_1 w_H - \lambda_2 \frac{\partial g}{\partial t_H} = 0 \tag{A.5}$$

$$c_H : \phi_H \frac{\partial u_H}{\partial c_H} - \lambda_1 = 0 \tag{A.6}$$

$$m : \lambda_2 - \lambda_1 + \mu = 0 \tag{A.7}$$

$$c_K : (\phi_H \delta_H + \phi_W \delta_W) \frac{\partial u_K}{\partial c_K} - \lambda_1 = 0 \tag{A.8}$$

where λ_1 and λ_2 are the Lagrange Multipliers for the budget constraint (3) and the childcare technology constraint (1), respectively, and μ is the Kuhn-Tucker Multiplier for the nonnegativity constraint (2), with the following (dual and complementary slackness) conditions:

$$\mu m = 0 \text{ and } \mu \geq 0.$$

Part 1. Define $\lambda = \lambda_2$ and, from (A.7), write $\lambda + \mu = \lambda_1$. Then, from (A.2), we obtain:

$$\left(\frac{\lambda}{\mu + \lambda} \right) \cdot \frac{\partial g}{\partial t_W} + w_W \cdot \left(1 - \frac{\phi_W}{w_W (\lambda + \mu)} \frac{\partial u_W}{\partial t_W} \right) = 0$$

Define $\theta_W = \theta_W^* \cdot \nu$, where $\nu = 1 + \mu/\lambda$ and

$$\theta_W^* = \left(1 - \frac{\phi_W}{\lambda \nu w_W} \frac{\partial u_W}{\partial t_W}\right),$$

so that we obtain:

$$\frac{\partial g}{\partial t_W} + w_W \cdot \theta_W = 0. \tag{A.9}$$

Note that

$$\theta_W^* = 1 - \frac{\phi_W}{\lambda \nu w_W} \frac{\partial u_W}{\partial t_W} = 1 - \frac{\partial u_W / \partial t_W}{\partial u_W / \partial l_W} > 0,$$

using (A.1) and the assumptions on utility functions. Similarly, from (A.5), we obtain:

$$\frac{\partial g}{\partial t_H} + w_H \cdot \theta_H = 0. \tag{A.10}$$

This system of two equations (A.9)–(A.10) constitutes the first-order conditions of the optimization problem P_0 with the non-negativity constraint.

Part 2. We write: $\lambda_I = \lambda_1/\phi_I$. From (A.1)–(A.3) or (A.4)–(A.6), we obtain:

$$\frac{\partial u_I}{\partial l_I} - \lambda_I w_I = 0, \tag{A.11}$$

$$\frac{\partial u_I}{\partial t_I} - \lambda_I (1 - \theta_I) w_I = 0, \tag{A.12}$$

$$\frac{\partial u_I}{\partial c_I} - \lambda_I = 0, \tag{A.13}$$

with $I = W$ and H. This system of three equations (A.11)–(A.13) constitutes the first-order conditions of the optimization problem P_I, where λ_I is the Lagrange Multiplier of the budget constraint.□

Proof of Proposition 2.

From the first-order conditions of Problem P_0, we obtain:

$$\theta_I w_I + \frac{\partial g}{\partial t_I}(t_W^*, t_H^*, n) = 0 \tag{A.14}$$

with $I = W$ and H. Replacing θ_I by $\nu(1-\varphi_I'(t_I^*))$, where ν is a Kuhn-Tucker multiplier defined in the Proof of Proposition 1 gives:

$$\nu(1-\varphi_H'(t_H^*))w_H + \frac{\partial g}{\partial t_H}(t_W^*, t_H^*, n) = 0, \tag{A.15}$$

$$\nu(1-\varphi_W'(t_W^*))w_W + \frac{\partial g}{\partial t_W}(t_W^*, t_H^*, n) = 0, \tag{A.16}$$

with

$$\nu g(t_W^*, t_H^*, n) = 0 \text{ and } \nu \geq 0, \tag{A.17}$$

This system of three equations (A.15)–(A.17) can be solved with respect to t_W^*, t_H^* and ν. From the first-order conditions of Problems P_I with respect to l_I, t_I and c_I, respectively, we also obtain:

$$\frac{\partial u_I}{\partial L_I}(L_I^*, c_I^*) = \lambda_I w_I,$$

$$\frac{\partial u_I}{\partial L_I}(L_I^*, c_I^*) \cdot (1-\nu(1-\varphi_I'(t_I^*))) = \lambda_I(1-\theta_I)w_I,$$

$$\frac{\partial u_I}{\partial c_I}(L_I^*, c_I^*) = \lambda_I,$$

where λ_I is the Lagrange Multiplier of the spouse's budget constraint. If $1-\nu(1-\varphi_I'(t_I^*) = 1-\theta_I$, the second first-order condition is redundant and can be eliminated. The budget constraint $\kappa_I - c_I - l_I w_I - t_I(1-\theta_I)w_I \geq 0$ is then written as:

$$\kappa_I - c_I - L_I w_I + \varphi_I(t_I^*)w_I - t_I^*(1-\theta_I)w_I \geq 0.$$

Using this budget constraint and defining $\pi_I = \varphi_I(t_I^*)w_I - t_I^*(1-\theta_I)w_I$, the first and the third first-order equations can be solved to give L_I^* and c_I^*, with $I = W$ and H. $\square$

Proof of Proposition 3.

The proof follows in two stages.

Part 1. Suppose first that (w_W, w_H, Y, n) is such that $m^*(w_W, w_H, Y, n) > 0$. Given that the childcare constraint is binding at equilibrium, we have:

$$m^*(w_W, w_H, Y, n) = g(t_W^*(w_W, w_H, Y, n), t_H^*(w_W, w_H, Y, n), n), \tag{A.18}$$

Differentiating both sides of this equation with respect to w_W and w_H, we obtain the following system of equations:

$$\frac{\partial m^*}{\partial w_W} = \frac{\partial g}{\partial t_W} \cdot \frac{\partial t_W^*}{\partial w_W} + \frac{\partial g}{\partial t_H} \cdot \frac{\partial t_H^*}{\partial w_W} \tag{A.19}$$

$$\frac{\partial m^*}{\partial w_H} = \frac{\partial g}{\partial t_W} \cdot \frac{\partial t_W^*}{\partial w_H} + \frac{\partial g}{\partial t_H} \cdot \frac{\partial t_H^*}{\partial w_H} \tag{A.20}$$

for any n. Since the function g is strictly convex, the determinant of the matrix

$$\begin{pmatrix} \frac{\partial t_W^*}{\partial w_W} & \frac{\partial t_H^*}{\partial w_W} \\ \frac{\partial t_W^*}{\partial w_H} & \frac{\partial t_H^*}{\partial w_H} \end{pmatrix} \tag{A.21}$$

is non-zero. Thus, this system of equations can be solved. Hence, we obtain:

$$\frac{\partial g}{\partial t_W} = a(w_W, w_H, Y, n)$$

$$\frac{\partial g}{\partial t_H} = b(w_W, w_H, Y, n)$$

where $a(w_W, w_H, Y, n)$ and $b(w_W, w_H, Y, n)$ are known functions. From this system of partial differential equations, the function $g\,(t_W, t_H, n)$ can be recovered up to an additive function of n. This remaining function can be determined by the boundary condition in (A.22). The prices of childcare time are then given by

$$\theta_W w_W = -\frac{\partial g}{\partial t_W}\left(t_W^*, t_H^*, n\right), \qquad \theta_H w_H = -\frac{\partial g}{\partial t_H}\left(t_W^*, t_H^*, n\right)$$

from the first-order conditions of the cost minimization problem.

Part 2. Suppose now (w_W, w_H, Y, n) is such that $m^*(w_W, w_H, Y, n) = 0$. In that case, the parental childcare time adjusts so that the childcare constraint is satisfied:

$$g(t_W^*(w_W, w_H, Y, n), t_H^*(w_W, w_H, Y, n), n) = 0, \tag{A.22}$$

for any w_H, w_W, Y, and n such that $m^*(w_W, w_H, Y, n) = 0$. The prices of childcare time can then be recovered by continuity at the boundary from the interior region. (note that the second equality can be seen as a restriction that childcare time functions must satisfy). □

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References

[1] Adda, Jérôme, Christian Dustmann, and Katrien Stevens(2017). "The Career Costs of Children". Journal of Political Economy, 125 (2), 293-337

[2] Apps, Patricia, and Ray Rees (2001). "Household production, full consumption and the costs of children." Labour Economics 8 (6), 621-648.

[3] Arduini, Francesca (2024). "Estimating intra-household sharing from time-use data". IFS Working Paper. London: Institute for Fiscal Studies.

[4] Bargain, Olivier, and Olivier Donni (2012). "Expenditure on children: A Rothbarth-type method consistent with scale economies and parents' bargaining." European Economic Review 56 (4), 792-813.

[5] Bargain, Olivier, Olivier Donni, and Imen Hentati (2022). “Resource sharing in households with children: A generalized model and empirical evidence from the UK." Journal of the European Economic Association 20 (6), 2468-2496.

[6] Bargain, Olivier, Guy Lacroix, and Luca Tiberti (2022). “Intrahousehold Resource Allocation and Individual Poverty: Assessing Collective Model Predictions using Direct Evidence on Sharing.” Economic Journal, 132 (643), 865-905.

[7] Benjamin, Dwayne (1992). “Household composition, labor markets, and labor demand: testing for separation in agricultural household models." Econometrica, 60 (2), 287-322.

[8] Bloemen, Hans, and Elena Stancanelli (2014). “Market hours, household work, child care, and wage rates of partners: an empirical analysis." Review of Economics of the Household, 12 (1), 51-81.

[9] Blundell, Richard, and Thomas MaCurdy (1999). “Labor supply: A review of alternative approaches." Handbook of Labor Economics 3, 1559-1695.

[10] Bradbury, Bruce (2008). “Time and the cost of children." Review of Income and Wealth 54 (3), 305-323.

[11] Brown, Caitlin, Rossella Calvi, and Jacob Penglase (2021). “Sharing the pie: An analysis of undernutrition and individual consumption in Bangladesh.” Journal of Public Economics 200, 104460.

[12] Budig, Michelle J., and Paula England (2001). “The wage penalty for motherhood." American Sociological Review 66 (2), 204-225.

[13] Calvi, Rossella, Jacob Penglase, Denni Tommasi, and Alexander Wolf (2023) “The more the poorer? Resource sharing and scale economies in large families.” Journal of Development Economics, 160, 102986.

[14] Chiappori, Pierre-André (1988). “Rational household labor supply." Econometrica, 56 (1), 63-90.

[15] Chiappori, Pierre-André (1992). “Collective labor supply and welfare." Journal of Political Economy 100 (3), 437-467.

[16] Chiappori, Pierre-Andre, Bernard Fortin, and Guy Lacroix (2002). “Marriage market, divorce legislation, and household labor supply." Journal of Political Economy 110 (1), 37-72.

[17] Colombino, Ugo (2000). “The cost of children when children are a choice." Labour 14 (1), 79-95.

[18] Cosaert, Sam, and Veerle Hennebel (2023). “Parental childcare with process benefits." Economica 90 (357), 339-371.

[19] Dechter, Evgenia Kogan (2014). “Maternity leave, effort allocation, and postmotherhood earnings." Journal of Human Capital 8 (2), 97-125.

[20] Diewert, W. E. (1971), “An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function”, Journal of Political Economy, 79 (3), 481-507.

[21] Donni, Olivier (2007). “Choix collectifs des heures travaillées, consommation publique et équilibre de Lindahl." Revue économique, 58 (3), 535-543.

[22] Donni, Olivier (2008). “Labor supply, home production, and welfare comparisons." Journal of Public Economics 92 (7), 1720-1737.

[23] Donni, Olivier (2009). “A simple approach to investigate intrahousehold allocation of private and public goods." The Review of Economics and Statistics 91 (3), 617-628.

[24] Donni, Olivier, and Pierre-André Chiappori (2011). “Nonunitary models of household behavior: a survey of the literature." In: Molina, J. Alberto (ed.). Household Economic Behaviors, Springer New York, NY.

[25] Donni, Olivier, and Eleonora Matteazzi (2018). “Collective decisions, household production, and labor force participation." Journal of Applied Econometrics 33 (7), 1064-1080.

[26] Donni, Olivier, and Anderson Vil (2024). "How to Incorporate Children into Labor Supply Equations. An Equivalence-Scale-Based Approach." mimeo.

[27] Dunbar, Geoffrey R., Arthur Lewbel, and Krishna Pendakur (2013). "Children's resources in collective households: identification, estimation, and an application to child poverty in Malawi." American Economic Review 103 (1), 438-471.

[28] Glauber, Rebecca (2018). "Trends in the motherhood wage penalty and fatherhood wage premium for low, middle, and high earners." Demography 55 (5), 1663-1680.

[29] Gørtz, Mette (2011). "Home production—Enjoying the process or the product?." Electronic International Journal of Time Use Research 8 (1), 85-109.

[30] Graham, John W., and Carole A. Green (1984). "Estimating the parameters of a household production function with joint products." The Review of Economics and Statistics, 66 (2), 277-282.

[31] Gronau, Reuben (1977). "Leisure, home production, and work—the theory of the allocation of time revisited." Journal of political economy 85 (6), 1099-1123.

[32] Gustafsson, Bjørn, and Urban Kjulin (1994). "Time use in child care and housework and the total cost of children." Journal of Population Economics, 7 (3), 287-306.

[33] Hallberg, Daniel, and Anders Klevmarken (2003). "Time for children: A study of parent's time allocation." Journal of Population Economics 16 (2), 205-226.

[34] Insolera, Noura E., David S. Johnson, and Beth A. Simmert (2019). "Evaluation of the Time Use data in PSID with comparisons to ATUS." Panel Study of Income Dynamics, Technical Series Paper #19-02.

[35] Juster, F. Thomas, and Frank P. Stafford (1991). "The allocation of time: Empirical findings, behavioral models, and problems of measurement." Journal of Economic literature, 29 (2), 471-522.

[36] Kalenkoski, Charlene M., David C. Ribar, and Leslie S. Stratton (2009). “The Influence of Wages on Parents’ Allocations of Time to Child Care and Market Work in the United Kingdom." Journal of Population Economics, 22 (2), 399-419

[37] Kerkhofs, Marcel, and Peter Kooreman (2003). “Identification and estimation of a class of household production models." Journal of Applied Econometrics, 18 (3), 337-369.

[38] Kimmel, J., & Connelly, R. (2007). “Mothers’ Time Choices: Caregiving, Leisure, Home Production, and Paid Work." The Journal of Human Resources, 42 (3), 643-681.

[39] Korenman, Sanders, and David Neumark (1992). “Marriage, motherhood, and wages." The Journal of Human Resources, 27 (2), 233-255.

[40] Koulovatianos, Christos, Carsten Schrder, and Ulrich Schmidt (2009). “Nonmarket household time and the cost of children." Journal of Business & Economic Statistics 27 (1), 42-51.

[41] Lechene, Valérie, Krishna Pendakur, and Alex Wolf (2022). “Ordinary least squares estimation of the intrahousehold distribution of expenditure.” Journal of Political Economy, 130 (3), 681-731.

[42] Lewbel, Arthur, and Krishna Pendakur (2024). “Estimating a model of inefficient cooperation and consumption in collective households”. Review of Economics of the Household, 22, 865–907.

[43] Lise, Jeremy, and Shannon Seitz (2011). “Consumption inequality and intra-household allocations." The Review of Economic Studies 78 (1), 328-355.

[44] Penglase, Jacob (2021). “Consumption inequality among children: Evidence from child fostering in Malawi.” Economic Journal 131 (634), 1000–1025.

[45] Stewart, Jay and Mary Dorinda Allard (2015). “Secondary Childcare in the ATUS: What Does It Measure?" IZA Discussion Paper No. 9376.

[46] Waldfogel, Jane (1998). “The family gap for young women in the United States and Britain: can maternity leave make a difference?" Journal of Labor Economics 16 (3), 505-545.